

Vortragsreihe

am: Dienstag, dem 20.11.2007

Zeit: 14.00 h

Ort: Hauptgebäude R 224.3

am: Mittwoch, dem 21.11.2007

Zeit: 10.00 h

Ort: Hauptgebäude R 224.3

am: Donnerstag, dem 22.11.2007

Zeit: 10.00 h

Ort: Hauptgebäude R 224.3

Es spricht: **Prof. Dr. J.F. Pommaret**
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über das Thema: **DIFFERENTIAL SEQUENCES AND
VARIATIONAL CALCULUS: FROM
ANALYTICAL MECHANICS TO
OPTIMAL CONTROL**

ABSTRACT

It is known that Lagrange himself was only considering variational calculus as a kind of recipe for finding equations, replacing questions on the “structure” of these equations by questions on how to exhibit a lagrangian function. Accordingly, it has been a challenge to study the following two “inverse problems” raised by the construction of Euler-Lagrange (EL) equations:

1) For a given first order lagrangian function, equating to zero the corresponding EL equations provides a second order system. The “first inverse problem” is to know whether a given second order system comes from a lagrangian function (J. Douglas, 1940).

2) For a given first order lagrangian function, the right members of the EL equations, also called “generalized force” after the Lorentz force in electromagnetism, are second order expressions. The “second inverse problem” is to know whether a given set of second order expressions can be considered as right members of convenient EL equations. Differential conditions, known as “Helmholtz conditions”, give the answer.

High order generalizations have been proposed, in particular through the construction of the so-called “variational bicomplex” (T. Tsujishita, A.M. Vinogradov, W.M. Tulczyjew,...) dealing with infinite jets. A strictly computational limitation of the jet order has been obtained by D. Krupka in 1990. Extensions to variational calculus with constraints have also been proposed (I. Anderson,...).

Independently of this mechanical framework, we have noticed that, in classical optimal control theory, the control system constraining the cost functional integral is always implicitly assumed to be controllable though the reason for such an assumption is never explicitly stated in the literature. It is also now known that the possibility of parametrizing a given control system is equivalent to its controllability and is also a kind of inverse problem.

The purpose of these self-contained lectures is to revisit and unify all these inverse problems in the light of the formal theory of systems of partial differential (PD) equations and algebraic analysis. In particular, we shall relate these problems to the study of formal integrability and the constructions of differential sequences. Meanwhile, the Spencer cohomology will be crucially used in order to stay within a given order of the jets involved.

Many explicit examples will illustrate the duality techniques involved through the systematic use of adjoint operators in PD optimal control, that is PD variational control with PD constraints.

Bibliography: J.-F. Pommaret, “Partial differential control theory”, Kluwer, 2001, 2 vol, 1000 pp, in particular Chapter VI(p 885-937).

Wir laden herzlich alle Interessierten zu diesem Vortrag ein.