Lehrstuhl B für Mathematik
Rheinisch-Westfälische Technische Hochschule
Prof. Dr. Wilhelm Plesken

Aachen, den 04.11.2011

Vortrag

am:	Freitag, dem 11.11.2011
Zeit:	15.00 h
Ort:	Sammelbau, Raum 329
Es spricht:	**Prof. Dr. Ihsen Yengui** **University of Sfax, Tunesien**
über das Thema:	**Algorithms for computing syzygies over $\mathbf{V}[X_1, \ldots, X_k]$, $\mathbf{V}$ a valuation ring**

ABSTRACT

The following new result will be presented: a valuation domain $\mathbf{V}$ has Krull dimension ≤ 1 if and only if for every finitely generated ideal I of $\mathbf{V}[X_1, \ldots, X_n]$, fixing the lexicographic order as monomial order, the ideal generated by the leading terms of the elements of I is also finitely generated. This proves the Gröbner ring conjecture. The same result is valid for Prüfer domains.

As a "scoop", contrary to the common idea that Gröbner bases can be computed exclusively on Noetherian ground, we prove that computing Gröbner bases over $\mathbf{R}[X_1, \ldots, X_n]$, where $\mathbf{R}$ is a Prüfer domain, has nothing to do with Noetherianity, it is only related to the fact that the Krull dimension of $\mathbf{R}$ is ≤ 1 opening the doors to a wider class of rings over which Gröbner bases can be computed (the class of Prüfer domains of Krull dimension ≤ 1 instead of that of Dedekind domains).

Moreover, I will give a general algorithm for computing a finite generating set for the syzygies of any finitely generated ideal of $\mathbf{V}[X_1, \ldots, X_k]$ ($\mathbf{V}$ a valuation domain) which neither relies on Noetherianity nor on Krull dimension. I will in fact give an algorithm for computing a finite generating set for the $\mathbf{V}$-saturation of any finitely generated submodule of $\mathbf{V}[X_1, \ldots, X_k]^n$ (this general result we establish seems to be new as we find no record to it in the literature). This algorithm is based on a notion of "echelon form" which ensure its correctness. Its termination proof is only combinatorial. Computing syzygies over $\mathbf{V}[X_1, \ldots, X_k]$ is one important application of the saturation algorithm we give.

Wir laden herzlich alle Interessierten zu diesem Vortrag ein.