Introduction to the DifferentialThomas package

Description

- The DifferentialThomas package provides functions to perform a disjoint decomposition of a system of differential equations and inequations into so-called simple and integrable systems. The definition of simple and integrable systems and the algorithm are derived from Joseph Miller Thomas' work.

Examples

```
> with(DifferentialThomas);
```

[*CallJets*, *CheckResult*, *CheckResults*, *ClearRememberData*, *ComparePolyInfo*, *ComputeRanking*, *ComputeWithDiffalg*, *Diff2JetList*, *DifferentialSystemCountingPolynomial*, *DifferentialSystemEquations*, *DifferentialSystemFactorModuleBasis*, *DifferentialSystemHilbertSeries*, *DifferentialSystemInequations*, *DifferentialSystemNormalForm*, *DifferentialSystemReduce*, *DifferentialThomasDecomposition*, *DifferentialThomasNewOptions*, *DifferentialThomasOptions*, *Initial*, *JetList2Diff*, *Leader*, *LeadingDerivation*, *LeadingFunction*, *MyPDSolve*, *PartialDerivative*, *PowerSeriesSolution*, *PrettyPrintDifferentialSystem*, *PrintDifferentialSystem*, *PrintDifferentialSystemJanetTrees*, *PrintDifferentialSystemJanetTreesCombinatorics*, *PrintRanking*, *Rank*, *Separant*, *SimplifyInequationsInDifferentialSystem*, *StandardForm*, *SystemTime*, *Walk*] **(2.1)**

Set the independent variables:

```
> ivar:=[x,y]:
```

Set the dependent variables

```
> dvar:=[u,v]:
```

Tell maple to do all computations with above variables and the ranking with u>>v:

```
> ComputeRanking(ivar,dvar,"EliminateFunction"):
```

The equations to compute with:

```
> L:=[u[1,0]^2-2*u[1,0]*v[0,1]+v[0,1]^2, u[0,1]^2+2*u[0,1]*v
  [1,0]+v[1,0]^2]:
  JetList2Diff(L);
```

$$\left[\left(\frac{\partial}{\partial x} u(x, y)\right)^2 - 2\left(\frac{\partial}{\partial x} u(x, y)\right)\left(\frac{\partial}{\partial y} v(x, y)\right) + \left(\frac{\partial}{\partial y} v(x, y)\right)^2, \left(\frac{\partial}{\partial y} u(x, y)\right)^2 + 2\left(\frac{\partial}{\partial y} u(x, y)\right)\left(\frac{\partial}{\partial x} v(x, y)\right) + \left(\frac{\partial}{\partial x} v(x, y)\right)^2\right] \quad \textbf{(2.2)}$$

Take the equations from L and compute a decomposition without inequations:

```
> res:=DifferentialThomasDecomposition(L,[]);
```

$$res := [DifferentialSystem] \quad (2.3)$$

We get only one resulting system (see PrintDifferentialSystem):

```
> PrintDifferentialSystem(res[1]);
Equations:
u:
u[1,0], [inf,inf,], 1, u[1,0]-v[0,1]
u[0,1], [  0,inf,], 1, u[0,1]+v[1,0]
v:
v[2,0], [inf,inf,], 1, v[2,0]+v[0,2]
Inequations:
```

```
> DifferentialSystemEquations(res[1]);
  JetList2Diff(%);
```

$$[u_{1,0} - v_{0,1}, u_{0,1} + v_{1,0}, v_{2,0} + v_{0,2}] \quad (2.4)$$

$$\left[\frac{\partial}{\partial x} u(x,y) - \left(\frac{\partial}{\partial y} v(x,y)\right), \frac{\partial}{\partial y} u(x,y) + \frac{\partial}{\partial x} v(x,y), \frac{\partial^2}{\partial x^2} v(x,y) + \frac{\partial^2}{\partial y^2} v(x,y)\right]$$

```
> DifferentialSystemInequations(res[1]);
  JetList2Diff(%);
```

$$[\,] \quad (2.5)$$

$$[\,]$$

We are interested in solutions for this system:

```
> PowerSeriesSolution(res[1],5,[0,0]);
```

$$\left[u_{0,0} + v_{0,1}x - v_{1,0}y + \frac{1}{2}v_{1,1}x^2 + v_{0,2}yx - \frac{1}{2}v_{1,1}y^2 - \frac{1}{6}v_{0,3}x^3 + \frac{1}{2}v_{1,2}yx^2 + \frac{1}{2}v_{0,3}y^2x - \frac{1}{6}v_{1,2}y^3 - \frac{1}{24}v_{1,3}x^4 - \frac{1}{6}v_{0,4}yx^3 + \frac{1}{4}v_{1,3}y^2x^2 + \frac{1}{6}v_{0,4}y^3x - \frac{1}{24}v_{1,3}y^4, v_{0,0} + v_{1,0}x + v_{0,1}y - \frac{1}{2}v_{0,2}x^2 + v_{1,1}yx + \frac{1}{2}v_{0,2}y^2 - \frac{1}{6}v_{1,2}x^3 - \frac{1}{2}v_{0,3}yx^2 + \frac{1}{2}v_{1,2}y^2x + \frac{1}{6}v_{0,3}y^3 + \frac{1}{24}v_{0,4}x^4 - \frac{1}{6}v_{1,3}yx^3 - \frac{1}{4}v_{0,4}y^2x^2 + \frac{1}{6}v_{1,3}y^3x + \frac{1}{24}v_{0,4}y^4\right] \quad (2.6)$$

This formal power series does not give convergent power series because of our ranking. But we can see the "factormodulebasis" in the coefficients:

```
> DifferentialSystemFactorModuleBasis(res[1]);
```

$$\left[1, -\frac{x+1}{-1+y}\right] \quad (2.7)$$

A differential system can be given to maples pdsolve directly and again one recognises two single parameter functions and one constant to be arbitrary.

```
> MyPDSolve(res[1]);
```

(2.8)

$$\{u(x,y) = \_F1(y - \mathrm{I}\,x)\,\mathrm{I} - \mathrm{I}\,\_F2(y + x\,\mathrm{I}) + \_C1,\ v(x,y) = \_F1(y - \mathrm{I}\,x) + \_F2(y + x\,\mathrm{I})\} \quad (2.8)$$

We want to check, whether all solutions of the system vanish on p:

```
> p:=diff(u(x,y),x)*diff(u(x,y),y)+diff(u(x,y),x)*diff(v(x,y),
  x)-diff(v(x,y),y)*diff(v(x,y),x):
```

```
> DifferentialSystemNormalForm(res[1],Diff2JetList(p));
```

$$-v_{0,1}\,v_{1,0} \quad (2.9)$$

This in not the case.

```
> q:=Diff2JetList(p);
```

$$q := u_{1,0}\,u_{0,1} + u_{1,0}\,v_{1,0} - v_{0,1}\,v_{1,0} \quad (2.10)$$

The package furthermore supports a few basic operations on polynomials. The leader is the highest appearing variable in the polynom with respect to the ranking.

```
> Leader(q);
```

$$u_{1,0} \quad (2.11)$$

The rank is the degree of the leader:

```
> Rank(q);
```

$$1 \quad (2.12)$$

The initial is the coefficient of the leader in the rank:

```
> Initial(q);
  collect(q,Leader(q));
```

$$u_{0,1} + v_{1,0} \quad (2.13)$$

$$\left(u_{0,1} + v_{1,0}\right)\,u_{1,0} - v_{0,1}\,v_{1,0}$$

We can compute partial derivations of polynomials:

```
> PartialDerivative(q,x);
  PartialDerivative(q,y);
```

$$u_{2,0}\,u_{0,1} + u_{1,0}\,u_{1,1} + u_{2,0}\,v_{1,0} + u_{1,0}\,v_{2,0} - v_{1,1}\,v_{1,0} - v_{0,1}\,v_{2,0} \quad (2.14)$$

$$u_{1,1}\,u_{0,1} + u_{1,0}\,u_{0,2} + u_{1,1}\,v_{1,0} + u_{1,0}\,v_{1,1} - v_{0,2}\,v_{1,0} - v_{0,1}\,v_{1,1}$$

See Also

with, DifferentialSystemReduce, ComputeRanking

Reduction in the DifferentialThomas package

Calling Sequence:

DifferentialSystemReduce(system,p)
DifferentialSystemReduce(p,system)
DifferentialSystemReduce(system,l2)
DifferentialSystemReduce(l2,system)
DifferentialSystemReduce(l1,p)
DifferentialSystemReduce(p,l1)
DifferentialSystemReduce(l1,l2)
DifferentialSystemReduce(l2,l1)
DifferentialSystemNormalForm(system,p)
DifferentialSystemNormalForm(p,system)
DifferentialSystemNormalForm(system,l2)
DifferentialSystemNormalForm(l2,system)
DifferentialSystemNormalForm(l1,p)
DifferentialSystemNormalForm(p,l1)
DifferentialSystemNormalForm(l1,l2)
DifferentialSystemNormalForm(l2,l1)

Parameters:

system - a differential system
p - a differential polynomial
l1 - list of differential systems
l2 - list of differential polynomials

Description

DifferentialSystemReduce and DifferentialSystemNormalForm reduce the given (list of) differential polynomial(s) with respect to the (list of) differential system (s).

The output is a (list of (lists of)) differential polynomial(s).

DifferentialSystemReduce applies a pseudo reduction algorithm. This means that the resulting polynomial is equivalent to the input polynomial up to a factor.

DifferentialSystemNormalForm applies a pseudo reduction algorithm but keeps track of denominators. This means that the result is equivalent to the input polynomial, but might be a fraction of differential polynomials.

Examples

```
> with(DifferentialThomas):
```

Set the independent variables:

```
> ivar:=[t]:
```

Set the dependent variables

```
> dvar:=[F,c1,sV,c]:
```

Tell maple to do all computations with above variables and the ranking with u>>v:

```
> ComputeRanking(ivar,dvar,"EliminateFunction"):
```

The equations to compute with:

```
> L:=[2*sV[1]*sV[0]-F[0]+k*sV[0], c[1]*sV[0]^2+2*sV[1]*sV[0]*c
  [0]-c1*F[0]+c[0]*k*sV[0], c1[1]]:
  L2:=[sV[0], F[0]]:
  map(a->(print@JetList2Diff)(a=0),L):
  map(a->(print@JetList2Diff)(a<>0),L2):
```

$$2\left(\frac{d}{dt}sV(t)\right)sV(t)-F(t)+k\,sV(t)=0 \quad \textbf{(2.1)}$$

$$\left(\frac{d}{dt}c(t)\right)sV(t)^2+2\left(\frac{d}{dt}sV(t)\right)sV(t)\,c(t)-c1(t)\,F(t)+c(t)\,k\,sV(t)=0$$

$$\frac{d}{dt}c1(t)=0$$

$$sV(t)\neq 0$$

$$F(t)\neq 0$$

```
> res:=DifferentialThomasDecomposition(L,L2);
```

$$res := [DifferentialSystem, DifferentialSystem] \quad \textbf{(2.2)}$$

We get two resulting system (see also PrintDifferentialSystem):

```
> map(print,PrettyPrintDifferentialSystem(res[1])):
```

$$2\left(\frac{d}{dt}sV(t)\right)sV(t)-F(t)+k\,sV(t)=0 \quad \textbf{(2.3)}$$

$$\left(\frac{d}{dt}c(t)\right)sV(t)-c1(t)\,k+2\left(\frac{d}{dt}sV(t)\right)c(t)+c(t)\,k-2\left(\frac{d}{dt}sV(t)\right)c1(t)=0$$

$$k\left(\frac{d^2}{dt^2}c(t)\right)sV(t)+5\,k\left(\frac{d}{dt}c(t)\right)\left(\frac{d}{dt}sV(t)\right)+\left(\frac{d}{dt}c(t)\right)k^2+2\left(\frac{d}{dt}sV(t)\right)\left(\frac{d^2}{dt^2}c(t)\right)sV(t)+6\left(\frac{d}{dt}c(t)\right)\left(\frac{d}{dt}sV(t)\right)^2-2\left(\frac{d^2}{dt^2}sV(t)\right)\left(\frac{d}{dt}c(t)\right)sV(t)=0$$

$$sV(t)\neq 0$$

$$\frac{d}{dt}c(t)\neq 0$$

$$k+2\left(\frac{d}{dt}sV(t)\right)\neq 0$$

```
> map(print,PrettyPrintDifferentialSystem(res[2])):
```

$$2\left(\frac{d}{dt}sV(t)\right)sV(t)-F(t)+k\,sV(t)=0 \quad \textbf{(2.4)}$$

$$-c1(t)+c(t)=0$$

$$\frac{d}{dt}c(t)=0$$

$$sV(t)\neq 0$$

$$k+2\left(\frac{d}{dt}sV(t)\right)\neq 0$$

Check whether all equations of the first system reduce to zero with respect to the second system:

```
> DifferentialSystemReduce(res[2],DifferentialSystemEquations
```

```
(res[1]));
```

$$[0, 0, 0] \quad (2.5)$$

See, how the elimination expresses F in term of the other differential variables:

```
> DifferentialSystemNormalForm(res[1],F[0]);
```

$$2\, sV_1\, sV_0 + k\, sV_0 \quad (2.6)$$

See Also

DifferentialThomas

Printing output systems in the DifferentialThomas package

Calling Sequence:

PrintDifferentialSystem(S)
PrintDifferentialSystemJanetTrees(S)
PrintDifferentialSystemJanetTreesCombinatorics(S)
PrettyPrintDifferentialSystem(S)

Parameters:

S - a differential System

Description

- PrintDifferentialSystem prints the differential systems in a way emphazising the underlying combinatorics of the Janet decomposition of differential Variables.
- This is quite useful to have an overview over the solution, even though the resulting equations and inequations are rather long.
- All output is done in jet-notation.
- First the equations are printed grouped by the dependent variable (differential indeterminate) underlying its leader. Each equations is printed having four entries:
  1) Its leader.
  2) The admissible prolongations, where an "inf" at the i-th position indicates that the i-th independent variable is admissible for prolongation, and a "0" indicates that it is not.
  3) The degree in its leader (=:rank).
  4) The equation.
- Then the inequations belonging to the system are printed. Each inequations is printed having three entries:
  1) Its leader.
  2) The degree in its leader (=:rank).
  3) The inequation.
- For only printing the equations the command PrintDifferentialSystemJanetTrees can be used.
- In the case of much to long output, one can supress printing the equations and just print the first three entries (1), (2) and (3) from above by using PrintDifferentialSystemJanetTreesCombinatorics.
- The procedure PrettyPrintDifferentialSystem prints a differential system by using Maples notations for differentiation. its output is given as a list of equations and inequations.

Examples

```
> restart;
> with(DifferentialThomas);
```

(2.1)

[*CallJets*, *CheckResult*, *CheckResults*, *ClearRememberData*, *ComparePolyInfo*, *ComputeRanking*, *ComputeWithDiffalg*, *Diff2JetList*, *DifferentialSystemCountingPolynomial*, *DifferentialSystemEquations*, *DifferentialSystemFactorModuleBasis*, *DifferentialSystemHilbertSeries*, *DifferentialSystemInequations*, *DifferentialSystemNormalForm*, *DifferentialSystemReduce*, *DifferentialThomasDecomposition*, *DifferentialThomasNewOptions*, *DifferentialThomasOptions*, *Initial*, *JetList2Diff*, *Leader*, *LeadingDerivation*, *LeadingFunction*, *MyPDSolve*, *PartialDerivative*, *PowerSeriesSolution*, *PrettyPrintDifferentialSystem*, *PrintDifferentialSystem*, *PrintDifferentialSystemJanetTrees*, *PrintDifferentialSystemJanetTreesCombinatorics*, *PrintRanking*, *Rank*, *Separant*, *SimplifyInequationsInDifferentialSystem*, *StandardForm*, *SystemTime*, *Walk*] **(2.1)**

(see ComputeRanking)

```
> ivar:=[x,y,z]:
  dvar:=[u,v,w]:
  ComputeRanking(ivar,dvar,"DegRevLex"):
```

Write down the equations:

```
> L:=[u[1,0,0]-2*u[1,0,0]*v[0,1,0]+v[0,1,0], u[0,1,0]*w[0,0,1]
  +2*u[0,1,0]*v[1,0,0]+v[1,0,0],w[0,0,0]-u[0,0,0]*u[0,1,0]]:
  JetList2Diff(L);
```

$$\left[\frac{\partial}{\partial x}u(x,y,z)-2\left(\frac{\partial}{\partial x}u(x,y,z)\right)\left(\frac{\partial}{\partial y}v(x,y,z)\right)+\frac{\partial}{\partial y}v(x,y,z),\left(\frac{\partial}{\partial y}u(x,y,z)\right)\left(\frac{\partial}{\partial z}w(x,y,z)\right)+2\left(\frac{\partial}{\partial y}u(x,y,z)\right)\left(\frac{\partial}{\partial x}v(x,y,z)\right)+\frac{\partial}{\partial x}v(x,y,z),w(x,y,z)-u(x,y,z)\left(\frac{\partial}{\partial y}u(x,y,z)\right)\right] \quad \textbf{(2.2)}$$

```
> res:=DifferentialThomasDecomposition(L,[]);
```

res := [*DifferentialSystem*, *DifferentialSystem*, *DifferentialSystem*, *DifferentialSystem*] **(2.3)**

```
> PrintDifferentialSystem(res[1]);
Equations:
u:
u[1,0,0], [inf,inf,inf,], 1, -u[1,0,0]-v[0,1,0]+2*u[1,0,0]*v[0,1,0]
u[0,1,0], [  0,inf,inf,], 1, -w[0,0,0]+u[0,0,0]*u[0,1,0]
v:
v[1,0,0], [inf,inf,inf,], 1, u[0,0,0]*v[1,0,0]+w[0,0,0]*w[0,0,1]+2*w[0,
0,0]*v[1,0,0]
v[0,2,0], [  0,inf,inf,], 1, u[0,0,0]*w[1,0,0]-4*u[0,0,0]*v[0,1,0]*w[1,
0,0]+u[0,0,0]^2*v[0,2,0]+4*u[0,0,0]*v[0,1,0]^2*w[1,0,0]+w[0,0,0]*v[0,1,
0]-2*w[0,0,0]*v[0,1,0]^2
w:
```

```
w[0,2,1], [inf,inf,inf,], 1, 8*w[0,0,0]^2*w[0,0,1]*u[0,0,0]^2*v[0,1,0]*
w[1,0,0]-2*w[0,0,0]*w[0,0,1]*u[0,0,0]^4*w[0,2,0]+2*w[0,0,0]*w[0,0,1]*u
[0,0,0]^3*w[0,1,0]+24*v[0,1,0]*w[0,0,0]^2*u[0,0,0]^2*w[1,0,0]-48*u[0,0,
0]^2*v[0,1,0]^2*w[1,0,0]*w[0,0,0]^2+16*w[0,0,0]^2*w[0,1,1]*u[0,0,0]^3*w
[1,0,0]+16*u[0,0,0]^2*v[0,1,0]*w[0,0,0]^3*w[0,1,1]-48*w[0,0,0]^2*u[0,0,
0]^3*v[0,1,0]*w[2,0,0]+48*w[0,0,0]^2*v[0,1,0]^2*u[0,0,0]^3*w[2,0,0]-4*w
[0,0,0]*w[0,1,0]*u[0,0,0]^4*w[0,1,1]-4*w[0,1,0]*w[0,0,1]*w[0,0,0]^2*u
[0,0,0]^2+16*w[0,0,0]^4*v[0,1,0]^2+16*w[0,0,0]^3*w[0,1,1]*u[0,0,0]^2*w
[1,0,0]+16*u[0,0,0]*v[0,1,0]*w[0,0,0]^4*w[0,1,1]-32*w[0,0,0]^3*u[0,0,0]
^2*v[0,1,0]*w[2,0,0]+32*w[0,0,0]^3*v[0,1,0]^2*u[0,0,0]^2*w[2,0,0]-8*u
[0,0,0]^4*v[0,1,0]*w[1,0,0]*w[0,1,0]*w[0,0,1]-8*u[0,0,0]^4*v[0,1,0]*w
[1,0,0]*w[0,0,0]*w[0,1,1]+4*w[0,0,0]*v[0,1,0]*w[0,1,0]*w[0,0,1]*u[0,0,
0]^3-4*w[0,0,0]^3*w[0,0,1]*u[0,0,0]*v[0,1,0]-8*w[0,0,0]^3*w[0,0,1]*u[0,
0,0]*w[1,0,0]+4*w[0,0,0]^2*v[0,1,0]*w[0,1,1]*u[0,0,0]^3+4*w[0,1,0]*w[0,
0,1]*u[0,0,0]^4*w[1,0,0]+4*w[0,0,0]*w[0,1,1]*u[0,0,0]^4*w[1,0,0]+24*u
[0,0,0]^4*w[0,0,0]*v[0,1,0]^2*w[2,0,0]-24*v[0,1,0]*u[0,0,0]^4*w[0,0,0]*
w[2,0,0]+12*w[0,0,0]^2*v[0,1,0]^2*u[0,0,0]^2-8*w[0,0,0]^4*w[0,0,1]*v[0,
1,0]+16*w[0,0,0]^3*w[0,0,1]*u[0,0,0]*v[0,1,0]*w[1,0,0]+8*w[0,0,0]^2*w
[0,0,1]*v[0,1,0]*w[0,1,0]*u[0,0,0]^2-32*u[0,0,0]*w[0,0,0]^3*w[1,0,0]*v
[0,1,0]^2-32*w[0,0,0]^3*w[1,0,0]*v[0,1,0]*w[0,1,1]*u[0,0,0]^2+u[0,0,0]
^5*w[2,0,0]+16*v[0,1,0]*u[0,0,0]*w[1,0,0]*w[0,0,0]^3-4*u[0,0,0]*w[0,0,
0]^4*w[0,1,1]-4*w[0,0,0]^3*w[0,2,1]*u[0,0,0]^3+8*w[0,0,0]^3*u[0,0,0]^2*
w[2,0,0]+4*u[0,0,0]^5*v[0,1,0]^2*w[2,0,0]+2*u[0,0,0]^3*w[0,0,0]*v[0,1,
0]^2-4*v[0,1,0]*u[0,0,0]^5*w[2,0,0]-24*u[0,0,0]^3*v[0,1,0]^2*w[1,0,0]*w
[0,0,0]+12*u[0,0,0]^3*w[0,0,0]*v[0,1,0]*w[1,0,0]+2*u[0,0,0]^4*v[0,1,0]*
w[1,0,0]-4*u[0,0,0]^4*v[0,1,0]^2*w[1,0,0]+w[0,0,0]^2*w[0,1,1]*u[0,0,0]
^3-u[0,0,0]^5*w[0,2,0]*w[0,0,1]-2*w[0,1,0]*u[0,0,0]^5*w[0,1,1]-u[0,0,0]
^5*w[0,0,0]*w[0,2,1]+6*w[0,0,0]*u[0,0,0]^4*w[2,0,0]-4*u[0,0,0]^4*w[0,0,
0]^2*w[0,2,1]+12*w[0,0,0]^2*u[0,0,0]^3*w[2,0,0]+24*u[0,0,0]*v[0,1,0]^2*
w[0,0,0]^3-16*w[0,0,0]*v[0,1,0]*w[0,0,1]*u[0,0,0]^3*w[1,0,0]*w[0,1,0]
+8*w[0,0,0]*w[0,0,1]*u[0,0,0]^3*w[1,0,0]*w[0,1,0]-32*w[0,0,0]^2*w[0,1,
1]*w[1,0,0]*u[0,0,0]^3*v[0,1,0]-2*u[0,0,0]*w[0,0,1]*w[0,0,0]^3-4*w[0,0,
0]^2*w[0,0,1]*u[0,0,0]^2*w[1,0,0]+4*w[0,0,1]*u[0,0,0]^4*w[0,1,0]^2
Inequations:
w[0,0,0], 1: w[0,0,0]
u[0,0,0], 1: u[0,0,0]
u[0,0,0], 1: u[0,0,0]+2*w[0,0,0]
v[0,1,0], 1: 2*v[0,1,0]-1
```

```
> PrintDifferentialSystemJanetTrees(res[1]);
```

```
u:
u[1,0,0], [inf,inf,inf,], 1, -u[1,0,0]-v[0,1,0]+2*u[1,0,0]*v[0,1,0]
u[0,1,0], [  0,inf,inf,], 1, -w[0,0,0]+u[0,0,0]*u[0,1,0]
v:
v[1,0,0], [inf,inf,inf,], 1, u[0,0,0]*v[1,0,0]+w[0,0,0]*w[0,0,1]+2*w[0,
0,0]*v[1,0,0]
v[0,2,0], [  0,inf,inf,], 1, u[0,0,0]*w[1,0,0]-4*u[0,0,0]*v[0,1,0]*w[1,
0,0]+u[0,0,0]^2*v[0,2,0]+4*u[0,0,0]*v[0,1,0]^2*w[1,0,0]+w[0,0,0]*v[0,1,
0]-2*w[0,0,0]*v[0,1,0]^2
w:
w[0,2,1], [inf,inf,inf,], 1, 8*w[0,0,0]^2*w[0,0,1]*u[0,0,0]^2*v[0,1,0]*
w[1,0,0]-2*w[0,0,0]*w[0,0,1]*u[0,0,0]^4*w[0,2,0]+2*w[0,0,0]*w[0,0,1]*u
[0,0,0]^3*w[0,1,0]+24*v[0,1,0]*w[0,0,0]^2*u[0,0,0]^2*w[1,0,0]-48*u[0,0,
0]^2*v[0,1,0]^2*w[1,0,0]*w[0,0,0]^2+16*w[0,0,0]^2*w[0,1,1]*u[0,0,0]^3*w
[1,0,0]+16*u[0,0,0]^2*v[0,1,0]*w[0,0,0]^3*w[0,1,1]-48*w[0,0,0]^2*u[0,0,
0]^3*v[0,1,0]*w[2,0,0]+48*w[0,0,0]^2*v[0,1,0]^2*u[0,0,0]^3*w[2,0,0]-4*w
[0,0,0]*w[0,1,0]*u[0,0,0]^4*w[0,1,1]-4*w[0,1,0]*w[0,0,1]*w[0,0,0]^2*u
[0,0,0]^2+16*w[0,0,0]^4*v[0,1,0]^2+16*w[0,0,0]^3*w[0,1,1]*u[0,0,0]^2*w
[1,0,0]+16*u[0,0,0]*v[0,1,0]*w[0,0,0]^4*w[0,1,1]-32*w[0,0,0]^3*u[0,0,0]
^2*v[0,1,0]*w[2,0,0]+32*w[0,0,0]^3*v[0,1,0]^2*u[0,0,0]^2*w[2,0,0]-8*u
[0,0,0]^4*v[0,1,0]*w[1,0,0]*w[0,1,0]*w[0,0,1]-8*u[0,0,0]^4*v[0,1,0]*w
[1,0,0]*w[0,0,0]*w[0,1,1]+4*w[0,0,0]*v[0,1,0]*w[0,1,0]*w[0,0,1]*u[0,0,
0]^3-4*w[0,0,0]^3*w[0,0,1]*u[0,0,0]*v[0,1,0]-8*w[0,0,0]^3*w[0,0,1]*u[0,
0,0]*w[1,0,0]+4*w[0,0,0]^2*v[0,1,0]*w[0,1,1]*u[0,0,0]^3+4*w[0,1,0]*w[0,
0,1]*u[0,0,0]^4*w[1,0,0]+4*w[0,0,0]*w[0,1,1]*u[0,0,0]^4*w[1,0,0]+24*u
[0,0,0]^4*w[0,0,0]*v[0,1,0]^2*w[2,0,0]-24*v[0,1,0]*u[0,0,0]^4*w[0,0,0]*
w[2,0,0]+12*w[0,0,0]^2*v[0,1,0]^2*u[0,0,0]^2-8*w[0,0,0]^4*w[0,0,1]*v[0,
```

```
1,0]+16*w[0,0,0]^3*w[0,0,1]*u[0,0,0]*v[0,1,0]*w[1,0,0]+8*w[0,0,0]^2*w
[0,0,1]*v[0,1,0]*w[0,1,0]*u[0,0,0]^2-32*u[0,0,0]*w[0,0,0]^3*w[1,0,0]*v
[0,1,0]^2-32*w[0,0,0]^3*w[1,0,0]*v[0,1,0]*w[0,1,1]*u[0,0,0]^2+u[0,0,0]
^5*w[2,0,0]+16*v[0,1,0]*u[0,0,0]*w[1,0,0]*w[0,0,0]^3-4*u[0,0,0]*w[0,0,
0]^4*w[0,1,1]-4*w[0,0,0]^3*w[0,2,1]*u[0,0,0]^3+8*w[0,0,0]^3*u[0,0,0]^2*
w[2,0,0]+4*u[0,0,0]^5*v[0,1,0]^2*w[2,0,0]+2*u[0,0,0]^3*w[0,0,0]*v[0,1,
0]^2-4*v[0,1,0]*u[0,0,0]^5*w[2,0,0]-24*u[0,0,0]^3*v[0,1,0]^2*w[1,0,0]*w
[0,0,0]+12*u[0,0,0]^3*w[0,0,0]*v[0,1,0]*w[1,0,0]+2*u[0,0,0]^4*v[0,1,0]*
w[1,0,0]-4*u[0,0,0]^4*v[0,1,0]^2*w[1,0,0]+w[0,0,0]^2*w[0,1,1]*u[0,0,0]
^3-u[0,0,0]^5*w[0,2,0]*w[0,0,1]-2*w[0,1,0]*u[0,0,0]^5*w[0,1,1]-u[0,0,0]
^5*w[0,0,0]*w[0,2,1]+6*w[0,0,0]*u[0,0,0]^4*w[2,0,0]-4*u[0,0,0]^4*w[0,0,
0]^2*w[0,2,1]+12*w[0,0,0]^2*u[0,0,0]^3*w[2,0,0]+24*u[0,0,0]*v[0,1,0]^2*
w[0,0,0]^3-16*w[0,0,0]*v[0,1,0]*w[0,0,1]*u[0,0,0]^3*w[1,0,0]*w[0,1,0]
+8*w[0,0,0]*w[0,0,1]*u[0,0,0]^3*w[1,0,0]*w[0,1,0]-32*w[0,0,0]^2*w[0,1,
1]*w[1,0,0]*u[0,0,0]^3*v[0,1,0]-2*u[0,0,0]*w[0,0,1]*w[0,0,0]^3-4*w[0,0,
0]^2*w[0,0,1]*u[0,0,0]^2*w[1,0,0]+4*w[0,0,1]*u[0,0,0]^4*w[0,1,0]^2
```

```
> PrintDifferentialSystemJanetTreesCombinatorics(res[1]);
```

```
u:
u[1,0,0], [inf,inf,inf,], 1
u[0,1,0], [  0,inf,inf,], 1
v:
v[1,0,0], [inf,inf,inf,], 1
v[0,2,0], [  0,inf,inf,], 1
w:
w[0,2,1], [inf,inf,inf,], 1
```

```
> PrettyPrintDifferentialSystem(res[1]);
```

$$\left[-\left(\frac{\partial}{\partial x} u(x,y,z)\right)-\left(\frac{\partial}{\partial y} v(x,y,z)\right)+2\left(\frac{\partial}{\partial x} u(x,y,z)\right)\left(\frac{\partial}{\partial y} v(x,y,z)\right)=0,\ -w(x,y,z)+u(x,y,z)\left(\frac{\partial}{\partial y} u(x,y,z)\right)=0,\ u(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)+w(x,y,z)\left(\frac{\partial}{\partial z} w(x,y,z)\right)+2\,w(x,y,z)\left(\frac{\partial}{\partial x} v(x,y,z)\right)=0,\ u(x,y,z)\left(\frac{\partial}{\partial x} w(x,y,z)\right)-4\,u(x,y,z)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right)+u(x,y,z)^2\left(\frac{\partial^2}{\partial y^2} v(x,y,z)\right)+4\,u(x,y,z)\left(\frac{\partial}{\partial y} v(x,y,z)\right)^2\left(\frac{\partial}{\partial x} w(x,y,z)\right)+w(x,y,z)\left(\frac{\partial}{\partial y} v(x,y,z)\right)-2\,w(x,y,z)\left(\frac{\partial}{\partial y} v(x,y,z)\right)^2=0,\right. \tag{2.4}$$

$$8\,w(x,y,z)^2\left(\frac{\partial}{\partial z} w(x,y,z)\right)u(x,y,z)^2\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right)-8\,u(x,y,z)^4\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)+4\,w(x,y,z)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial y} w(x,y,z)\right)\left(\frac{\partial}{\partial z} w(x,y,z)\right)u(x,y,z)^3+16\,w(x,y,z)^3\left(\frac{\partial}{\partial z} w(x,y,z)\right)u(x,y,z)\left(\frac{\partial}{\partial y} v(x,y,z)\right)\left(\frac{\partial}{\partial x} w(x,y,z)\right)+8\,w(x,y,z)^2\left(\frac{\partial}{\partial z} w(x,y,\right.$$

$$z)\Big)\left(\frac{\partial}{\partial y}v(x,y,z)\right)\left(\frac{\partial}{\partial y}w(x,y,z)\right)u(x,y,z)^2+8\,w(x,y,z)\Big(\frac{\partial}{\partial z}w(x,$$

$$y,z)\Big)u(x,y,z)^3\left(\frac{\partial}{\partial x}w(x,y,z)\right)\left(\frac{\partial}{\partial y}w(x,y,z)\right)-8\,u(x,y,$$

$$z)^4\left(\frac{\partial}{\partial y}v(x,y,z)\right)\left(\frac{\partial}{\partial x}w(x,y,z)\right)w(x,y,z)\left(\frac{\partial^2}{\partial z\,\partial y}w(x,y,z)\right)$$

$$-32\,w(x,y,z)^3\left(\frac{\partial}{\partial x}w(x,y,z)\right)\left(\frac{\partial}{\partial y}v(x,y,z)\right)\Big(\frac{\partial^2}{\partial z\,\partial y}w(x,y,$$

$$z)\Big)u(x,y,z)^2-32\,w(x,y,z)^2\left(\frac{\partial^2}{\partial z\,\partial y}w(x,y,z)\right)\Big(\frac{\partial}{\partial x}w(x,y,$$

$$z)\Big)u(x,y,z)^3\left(\frac{\partial}{\partial y}v(x,y,z)\right)-16\,w(x,y,z)\left(\frac{\partial}{\partial y}v(x,y,z)\right)\Big(\frac{\partial}{\partial z}w(x,$$

$$y,z)\Big)u(x,y,z)^3\left(\frac{\partial}{\partial x}w(x,y,z)\right)\left(\frac{\partial}{\partial y}w(x,y,z)\right)-4\,u(x,y,z)\,w(x,$$

$$y,z)^4\left(\frac{\partial^2}{\partial z\,\partial y}w(x,y,z)\right)+12\,w(x,y,z)^2\left(\frac{\partial}{\partial y}v(x,y,z)\right)^2u(x,y,z)^2$$

$$-8\,w(x,y,z)^4\left(\frac{\partial}{\partial z}w(x,y,z)\right)\left(\frac{\partial}{\partial y}v(x,y,z)\right)+2\,u(x,y,z)^3\,w(x,y,$$

$$z)\left(\frac{\partial}{\partial y}v(x,y,z)\right)^2+2\,u(x,y,z)^4\left(\frac{\partial}{\partial y}v(x,y,z)\right)\left(\frac{\partial}{\partial x}w(x,y,z)\right)$$

$$-4\,u(x,y,z)^4\left(\frac{\partial}{\partial y}v(x,y,z)\right)^2\left(\frac{\partial}{\partial x}w(x,y,z)\right)+24\,u(x,y,$$

$$z)\left(\frac{\partial}{\partial y}v(x,y,z)\right)^2w(x,y,z)^3-2\,u(x,y,z)\left(\frac{\partial}{\partial z}w(x,y,z)\right)w(x,y,z)^3$$

$$+4\left(\frac{\partial}{\partial z}w(x,y,z)\right)u(x,y,z)^4\left(\frac{\partial}{\partial y}w(x,y,z)\right)^2-4\,w(x,y,$$

$$z)^3\left(\frac{\partial^3}{\partial z\,\partial y^2}w(x,y,z)\right)u(x,y,z)^3+8\,w(x,y,z)^3\,u(x,y,z)^2\Big(\frac{\partial^2}{\partial x^2}w(x,y,$$

$$z)\Big)+4\,u(x,y,z)^5\left(\frac{\partial}{\partial y}v(x,y,z)\right)^2\left(\frac{\partial^2}{\partial x^2}w(x,y,z)\right)-4\Big(\frac{\partial}{\partial y}v(x,y,$$

$$z)\Big)u(x,y,z)^5\left(\frac{\partial^2}{\partial x^2}w(x,y,z)\right)+w(x,y,z)^2\left(\frac{\partial^2}{\partial z\,\partial y}w(x,y,z)\right)u(x,$$

$$y,z)^3-u(x,y,z)^5\left(\frac{\partial^2}{\partial y^2}w(x,y,z)\right)\left(\frac{\partial}{\partial z}w(x,y,z)\right)-2\Big(\frac{\partial}{\partial y}w(x,y,$$

$$z)\Big)u(x,y,z)^5\left(\frac{\partial^2}{\partial z\,\partial y}w(x,y,z)\right)-u(x,y,z)^5\,w(x,y,z)\Big(\frac{\partial^3}{\partial z\,\partial y^2}w(x,y,$$

$$z)\Big) + 6\, w(x, y, z)\, u(x, y, z)^4 \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right) - 4\, u(x, y, z)^4\, w(x, y,$$

$$z)^2 \left(\frac{\partial^3}{\partial z\, \partial y^2} w(x, y, z)\right) + 12\, w(x, y, z)^2\, u(x, y, z)^3 \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right)$$

$$+ 16\, w(x, y, z)^4 \left(\frac{\partial}{\partial y} v(x, y, z)\right)^2 + u(x, y, z)^5 \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right) + 2\, w(x,$$

$$y, z) \left(\frac{\partial}{\partial z} w(x, y, z)\right) u(x, y, z)^3 \left(\frac{\partial}{\partial y} w(x, y, z)\right) + 24 \Big(\frac{\partial}{\partial y} v(x, y,$$

$$z)\Big)\, w(x, y, z)^2\, u(x, y, z)^2 \left(\frac{\partial}{\partial x} w(x, y, z)\right) - 48\, u(x, y, z)^2 \Big(\frac{\partial}{\partial y} v(x,$$

$$y, z)\Big)^2 \left(\frac{\partial}{\partial x} w(x, y, z)\right) w(x, y, z)^2 - 4 \left(\frac{\partial}{\partial y} w(x, y, z)\right) \Big(\frac{\partial}{\partial z} w(x, y,$$

$$z)\Big)\, w(x, y, z)^2\, u(x, y, z)^2 - 4\, w(x, y, z)^3 \left(\frac{\partial}{\partial z} w(x, y, z)\right) u(x, y,$$

$$z) \left(\frac{\partial}{\partial y} v(x, y, z)\right) - 8\, w(x, y, z)^3 \left(\frac{\partial}{\partial z} w(x, y, z)\right) u(x, y, z) \Big(\frac{\partial}{\partial x} w(x, y,$$

$$z)\Big) + 4 \left(\frac{\partial}{\partial y} w(x, y, z)\right) \left(\frac{\partial}{\partial z} w(x, y, z)\right) u(x, y, z)^4 \left(\frac{\partial}{\partial x} w(x, y, z)\right)$$

$$- 32\, u(x, y, z)\, w(x, y, z)^3 \left(\frac{\partial}{\partial x} w(x, y, z)\right) \left(\frac{\partial}{\partial y} v(x, y, z)\right)^2$$

$$+ 16 \left(\frac{\partial}{\partial y} v(x, y, z)\right) u(x, y, z) \left(\frac{\partial}{\partial x} w(x, y, z)\right) w(x, y, z)^3 - 24\, u(x,$$

$$y, z)^3 \left(\frac{\partial}{\partial y} v(x, y, z)\right)^2 \left(\frac{\partial}{\partial x} w(x, y, z)\right) w(x, y, z) + 12\, u(x, y, z)^3\, w(x, y,$$

$$z) \left(\frac{\partial}{\partial y} v(x, y, z)\right) \left(\frac{\partial}{\partial x} w(x, y, z)\right) - 4\, w(x, y, z)^2 \Big(\frac{\partial}{\partial z} w(x, y,$$

$$z)\Big)\, u(x, y, z)^2 \left(\frac{\partial}{\partial x} w(x, y, z)\right) - 2\, w(x, y, z) \left(\frac{\partial}{\partial z} w(x, y, z)\right) u(x, y,$$

$$z)^4 \left(\frac{\partial^2}{\partial y^2} w(x, y, z)\right) + 16\, w(x, y, z)^2 \left(\frac{\partial^2}{\partial z\, \partial y} w(x, y, z)\right) u(x, y,$$

$$z)^3 \left(\frac{\partial}{\partial x} w(x, y, z)\right) + 16\, u(x, y, z)^2 \left(\frac{\partial}{\partial y} v(x, y, z)\right) w(x, y,$$

$$z)^3 \left(\frac{\partial^2}{\partial z\, \partial y} w(x, y, z)\right) - 48\, w(x, y, z)^2\, u(x, y, z)^3 \Big(\frac{\partial}{\partial y} v(x, y,$$

$$z)\Big) \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right) + 48\, w(x, y, z)^2 \left(\frac{\partial}{\partial y} v(x, y, z)\right)^2 u(x, y,$$

$$z)^3 \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right) - 4\, w(x, y, z) \left(\frac{\partial}{\partial y} w(x, y, z)\right) u(x, y,$$

$$z)^4 \left(\frac{\partial^2}{\partial z\, \partial y} w(x, y, z)\right) + 16\, w(x, y, z)^3 \left(\frac{\partial^2}{\partial z\, \partial y} w(x, y, z)\right) u(x, y,$$

$$z)^2 \left(\frac{\partial}{\partial x} w(x, y, z)\right) + 16\, u(x, y, z) \left(\frac{\partial}{\partial y} v(x, y, z)\right) w(x, y,$$

$$z)^4 \left(\frac{\partial^2}{\partial z\, \partial y} w(x, y, z)\right) - 32\, w(x, y, z)^3\, u(x, y, z)^2 \left(\frac{\partial}{\partial y} v(x, y,\right.$$

$$\left. z)\right) \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right) + 32\, w(x, y, z)^3 \left(\frac{\partial}{\partial y} v(x, y, z)\right)^2 u(x, y,$$

$$z)^2 \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right) + 4\, w(x, y, z)^2 \left(\frac{\partial}{\partial y} v(x, y, z)\right) \left(\frac{\partial^2}{\partial z\, \partial y} w(x, y,\right.$$

$$\left. z)\right) u(x, y, z)^3 + 4\, w(x, y, z) \left(\frac{\partial^2}{\partial z\, \partial y} w(x, y, z)\right) u(x, y, z)^4 \left(\frac{\partial}{\partial x} w(x, y,\right.$$

$$\left. z)\right) + 24\, u(x, y, z)^4\, w(x, y, z) \left(\frac{\partial}{\partial y} v(x, y, z)\right)^2 \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right)$$

$$- 24 \left(\frac{\partial}{\partial y} v(x, y, z)\right) u(x, y, z)^4\, w(x, y, z) \left(\frac{\partial^2}{\partial x^2} w(x, y, z)\right) = 0,\ w(x, y,$$

$$z) \neq 0,\ u(x, y, z) \neq 0,\ u(x, y, z) + 2\, w(x, y, z) \neq 0,\ 2 \left(\frac{\partial}{\partial y} v(x, y, z)\right) - 1$$

$$\neq 0]$$

```
> PrintDifferentialSystem(res[2]);
Equations:
u:
u[1,0,0], [inf,inf,inf,], 1, -u[1,0,0]-v[0,1,0]+2*u[1,0,0]*v[0,1,0]
u[0,1,0], [  0,inf,inf,], 1, u[0,1,0]
v:
v[1,0,0], [inf,inf,inf,], 1, v[1,0,0]
v[0,2,0], [  0,inf,inf,], 1, v[0,2,0]
w:
w[0,0,0], [inf,inf,inf,], 1, w[0,0,0]
Inequations:
u[0,0,0], 1: u[0,0,0]
v[0,1,0], 1: 2*v[0,1,0]-1
```

```
> PrintDifferentialSystem(res[3]);
Equations:
u:
u[0,0,0], [inf,inf,inf,], 1, u[0,0,0]+2*w[0,0,0]
v:
v[0,2,0], [inf,inf,inf,], 1, v[0,2,0]
v[0,1,1], [inf,  0,inf,], 1, v[0,1,1]
w:
w[1,0,0], [inf,inf,inf,], 1, v[0,1,0]-2*w[1,0,0]+4*v[0,1,0]*w[1,0,0]
w[0,1,0], [  0,inf,inf,], 1, -1+4*w[0,1,0]
w[0,0,1], [  0,  0,inf,], 1, w[0,0,1]
Inequations:
w[0,0,0], 1: w[0,0,0]
```

```
v[0,1,0], 1: 2*v[0,1,0]-1
```

```
> PrintDifferentialSystem(res[4]);
```

```
Equations:
u:
u[0,0,0], [inf,inf,inf,], 1, u[0,0,0]
v:
v[1,0,0], [inf,inf,inf,], 1, v[1,0,0]
v[0,1,0], [  0,inf,inf,], 1, v[0,1,0]
w:
w[0,0,0], [inf,inf,inf,], 1, w[0,0,0]
Inequations:
```

See Also

DifferentialThomas, ComputeRanking

Rankings in the DifferentialThomas package

Calling Sequence:

ComputeRanking(ivar,dvar,ranking,opt)

Parameters:

ivar - list of independet variables
dvar - list of dependet variables
ranking - the ranking (see below)
opt - (optional) string specifying options for the computation

Description

- Having an appropriate ranking is important for both speed of the differential thomas decomposition and for many applications.
- The ranking is globally set by ComputeRanking.
- The parameter ivar and dvar are given as a list of symbols. It is not possible to use "indexed" symbols.
- The Ranking itself in the parameter ranking has the following possibilities:
  1) "DegRevLex"
  2) "EliminateFunction": the earlier an element is in dvar, the smaller it is
  3) a list l of lists and/or symbols, where each list is nonempty and map(op,l) =dvar. For example l=[u1,[u2,u3],[u4],u5,[u6,u7,u8]] stands for u1>>u2, u3>>u4>>u5>>u6,u7,u8. This is called a block ranking (on the dependent variables).
  4) "Matrix"=A, where A is a left-invertible matrix with nops(ivar)+nops(dvar) columns. Furthermore in each column, the first nonzero entry of A has to be positive. So now assume, the you have two differential variables (e.g. u[2,3,4] and v[1,2,3]). Then each of these variables has an associated vector with nops (ivar)+nops(dvar) entries. The first nops(ivar) entries give the orders ("degree") of the differential variable in each independetn variable (e.g. [2,3,4] and [1,2,3]). Another entry is one if the differential variable is derived from the corresponding dvar and zero otherwise (e.g. for dvar=[u,v] we have <2,3,4,1,0> and <1,2,3,0,1>). These two vectors are each multiplied to A and the results are compared componentwise. This actually gives any ranking.
- It is also possible to change between several rankings. This is possible by using "return" as an optional argument. This way one can have several rankings and give them to DifferentialThomasDecomposition as an optional argument.

Examples

```
> restart;
> with(DifferentialThomas);
```

[CallJets, CheckResult, CheckResults, ClearRememberData, ComparePolyInfo, ComputeRanking, ComputeWithDiffalg, Diff2JetList, DifferentialSystemCountingPolynomial, DifferentialSystemEquations, DifferentialSystemFactorModuleBasis, **(2.1)**

DifferentialSystemHilbertSeries, *DifferentialSystemInequations*, *DifferentialSystemNormalForm*, *DifferentialSystemReduce*, *DifferentialThomasDecomposition*, *DifferentialThomasNewOptions*, *DifferentialThomasOptions*, *Initial*, *JetList2Diff*, *Leader*, *LeadingDerivation*, *LeadingFunction*, *MyPDSolve*, *PartialDerivative*, *PowerSeriesSolution*, *PrettyPrintDifferentialSystem*, *PrintDifferentialSystem*, *PrintDifferentialSystemJanetTrees*, *PrintDifferentialSystemJanetTreesCombinatorics*, *PrintRanking*, *Rank*, *Separant*, *SimplifyInequationsInDifferentialSystem*, *StandardForm*, *SystemTime*, *Walk*]

Set the independent variables:

```
> ivar:=[x,y,z]:
```

Set the dependent variables

```
> dvar:=[u,v,w]:
```

Tell maple to do all computations with above variables and the ranking with u>>v>>w:

```
> ComputeRanking(ivar,dvar,"EliminateFunction"):
> p:=u[0,0,0]+v[1,0,1]+w[2,1,0];
```

$$p := u_{0,\,0,\,0} + v_{1,\,0,\,1} + w_{2,\,1,\,0} \tag{2.2}$$

```
> Leader(p);
```

$$u_{0,\,0,\,0} \tag{2.3}$$

We see, that the differential variable u[1,0] is higher then the other ones appering, even though it has much lower order. Now work with degree reverse lexicographical ordering:

```
> ComputeRanking(ivar,dvar,"DegRevLex");
> Leader(p);
```

$$w_{2,\,1,\,0} \tag{2.4}$$

Now we have an orderly ranking, so the differential variable with the highest order is the biggest one. Now we use a block ranking on the dependent variables:

```
> ComputeRanking(ivar,dvar,[[u,v],w]);
> Leader(p);
```

$$v_{1,\,0,\,1} \tag{2.5}$$

Since u and v are both in the highest block of the ranking, the one with higher order "wins".

Next thing is about keeping several rankings:

```
> RankingDegRevLex:=ComputeRanking(ivar,dvar,"DegRevLex",
  "return");
  RankingEliminateFunction:=ComputeRanking(ivar,dvar,
  "EliminateFunction","return");
  RankingBlock:=ComputeRanking(ivar,dvar,[[u,v],w],"return");
  A:=<<1,0,0,0,0,0>|<0,0,1,0,0,0>|<0,0,1,1,0,0>|<0,1,0,0,0,
```

```
0>|<0,1,0,0,1,0>|<0,0,0,0,0,1>>;
RankingMatrix:=ComputeRanking(ivar,dvar,"Matrix"=A,"return")
;
```

$$RankingDegRevLex := rankingtable \quad (2.6)$$

$$RankingEliminateFunction := rankingtable$$

$$RankingBlock := rankingtable$$

$$A := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RankingMatrix := rankingtable$$

The Matrix A corresponds to:
First look at order in x, if this is the same prefer a variable derived from u or v. If these are the same, take the total order in y and z and if these are the same prefer the higher order in z. If all these things are equal prefer v over u.

Now we can use the rankings:

```
> ComputeRanking(RankingDegRevLex):
  Leader(p);
```

$$w_{2,1,0} \quad (2.7)$$

```
> ComputeRanking(RankingEliminateFunction):
  Leader(p);
```

$$u_{0,0,0} \quad (2.8)$$

```
> ComputeRanking(RankingBlock):
  Leader(p);
```

$$v_{1,0,1} \quad (2.9)$$

```
> ComputeRanking(RankingMatrix):
  Leader(p);
```

$$w_{2,1,0} \quad (2.10)$$

Furthermore these ranking can be given to the decomposition algorithm, without changing the globally set ordering. At first compute with a globally set DegRevLex:

```
> ComputeRanking(RankingDegRevLex):
> L:=[u[1,0,0]^2-2*u[1,0,0]*v[0,1,0]+v[0,1,0]^2, u[0,1,0]^2+2*
  u[0,1,0]*v[1,0,0]+v[1,0,0]^2,p*u[0,0,0]*u[0,1,0]]:
  JetList2Diff(L);
```

$$\left[\left(\frac{\partial}{\partial x} u(x, y, z)\right)^2 - 2\left(\frac{\partial}{\partial x} u(x, y, z)\right)\left(\frac{\partial}{\partial y} v(x, y, z)\right) + \left(\frac{\partial}{\partial y} v(x, y, z)\right)^2, \quad (2.11)$$

$$\left(\frac{\partial}{\partial y} u(x, y, z)\right)^2 + 2\left(\frac{\partial}{\partial y} u(x, y, z)\right)\left(\frac{\partial}{\partial x} v(x, y, z)\right) + \left(\frac{\partial}{\partial x} v(x, y, z)\right)^2,$$

$$\left(u(x, y, z) + \frac{\partial^2}{\partial z\,\partial x} v(x, y, z) + \frac{\partial^3}{\partial y\,\partial x^2} w(x, y, z)\right) u(x, y, z) \left(\frac{\partial}{\partial y} u(x, y, z)\right)\Bigg]$$

```
> res:=DifferentialThomasDecomposition(L,[]);
```

$res := [DifferentialSystem, DifferentialSystem, DifferentialSystem]$ **(2.12)**

Then we can use other orderings:

```
> res:=DifferentialThomasDecomposition(L,[],RankingBlock);
```

$res := [DifferentialSystem, DifferentialSystem, DifferentialSystem, DifferentialSystem]$ **(2.13)**

```
> res:=DifferentialThomasDecomposition(L,[],RankingMatrix);
```

$res := [DifferentialSystem, DifferentialSystem, DifferentialSystem]$ **(2.14)**

```
> res:=DifferentialThomasDecomposition(L,[],
  RankingEliminateFunction);
```

$res := [DifferentialSystem, DifferentialSystem, DifferentialSystem, DifferentialSystem]$ **(2.15)**

The globally set ranking can be printed with PrintRanking:

```
> PrintRanking();
The current Ranking for the indepedent variables [x, y, z] and the
dependent variables [u, v, w] is:
DegRevLex
```

This was the last ranking set as a global ranking.

See Also

DifferentialThomas