```
> restart:
> with(Janet):
```

Wir diskutieren das klassiche Beispiel von Janet:

```
> ivar:=[x,y,z];v:=op(ivar);
```

$$ivar := [x, y, z]$$

$$v := x, y, z$$

```
> dvar:=[u];
```

$$dvar := [u]$$

```
> L:=[diff(u(v),z,z)+y*diff(u(v),x,x), diff(u(v),y,y)];
```

$$L := \left[\left(\frac{\partial^2}{\partial z^2} \mathrm{u}(x, y, z)\right) + y \left(\frac{\partial^2}{\partial x^2} \mathrm{u}(x, y, z)\right), \frac{\partial^2}{\partial y^2} \mathrm{u}(x, y, z)\right]$$

```
> J:=JanetBasis(L,ivar,dvar);
```

$$J := \left[\left[\frac{\partial^2}{\partial y^2} \mathrm{u}(x, y, z), \left(\frac{\partial^2}{\partial z^2} \mathrm{u}(x, y, z)\right) + y \left(\frac{\partial^2}{\partial x^2} \mathrm{u}(x, y, z)\right), y \left(\frac{\partial^3}{\partial z^2 \partial y} \mathrm{u}(x, y, z)\right) - \left(\frac{\partial^2}{\partial z^2} \mathrm{u}(x, y, z)\right), \frac{\partial^3}{\partial y^2 \partial x} \mathrm{u}(x, y, z), \frac{\partial^4}{\partial z^4} \mathrm{u}(x, y, z), -\left(\frac{\partial^3}{\partial z^2 \partial x} \mathrm{u}(x, y, z)\right) + y \left(\frac{\partial^4}{\partial z^2 \partial y \partial x} \mathrm{u}(x, y, z)\right), \frac{\partial^5}{\partial z^4 \partial x} \mathrm{u}(x, y, z)\right], [x, y, z], [u]\right]$$

```
> TabVar();
```

$$\left[\frac{\partial^2}{\partial y^2} \mathrm{u}(x, y, z), [*, y, z], \frac{\partial^2}{\partial y^2} \mathrm{u}(x, y, z)\right]$$

$$\left[\left(\frac{\partial^2}{\partial z^2} \mathrm{u}(x, y, z)\right) + y \left(\frac{\partial^2}{\partial x^2} \mathrm{u}(x, y, z)\right), [x, y, z], y \left(\frac{\partial^2}{\partial x^2} \mathrm{u}(x, y, z)\right)\right]$$

$$\left[y \left(\frac{\partial^3}{\partial z^2 \partial y} \mathrm{u}(x, y, z)\right) - \left(\frac{\partial^2}{\partial z^2} \mathrm{u}(x, y, z)\right), [*, *, z], y \left(\frac{\partial^3}{\partial z^2 \partial y} \mathrm{u}(x, y, z)\right)\right]$$

$$\left[\frac{\partial^3}{\partial y^2 \partial x} \mathrm{u}(x, y, z), [*, y, z], \frac{\partial^3}{\partial y^2 \partial x} \mathrm{u}(x, y, z)\right]$$

$$\left[\frac{\partial^4}{\partial z^4} \mathrm{u}(x, y, z), [*, *, z], \frac{\partial^4}{\partial z^4} \mathrm{u}(x, y, z)\right]$$

$$\left[-\left(\frac{\partial^3}{\partial z^2 \partial x} \mathrm{u}(x, y, z)\right) + y \left(\frac{\partial^4}{\partial z^2 \partial y \partial x} \mathrm{u}(x, y, z)\right), [*, *, z], y \left(\frac{\partial^4}{\partial z^2 \partial y \partial x} \mathrm{u}(x, y, z)\right)\right]$$

$$\left[\frac{\partial^5}{\partial z^4 \partial x} \mathrm{u}(x, y, z), [*, *, z], \frac{\partial^5}{\partial z^4 \partial x} \mathrm{u}(x, y, z)\right]$$

```
> ZeroSets();
```

$$[[y, \{y = 0\}]]$$

Dies bedeutet, daß im Verlaufe des Algorithmus durch y geteilt wurde. Wir wissen also nur für $y \not = 0$ bescheid. Trotdem schauen wir hier, was uns der Output sagt:

```
> HilbertSeries(t);
```

$$1 + 3\,t + 4\,t^2 + 3\,t^3 + t^4$$

Wir bekommen eine genauere Information durch

```
> ParamDeriv(ivar,dvar);
```

$$\left[u(x,y,z), \frac{\partial}{\partial z}u(x,y,z), \frac{\partial}{\partial y}u(x,y,z), \frac{\partial}{\partial x}u(x,y,z), \frac{\partial^2}{\partial z^2}u(x,y,z), \frac{\partial^2}{\partial z\,\partial y}u(x,y,z), \frac{\partial^2}{\partial z\,\partial x}u(x,y,z), \frac{\partial^2}{\partial y\,\partial x}u(x,y,z), \frac{\partial^3}{\partial z^3}u(x,y,z), \frac{\partial^3}{\partial z^2\,\partial x}u(x,y,z), \frac{\partial^3}{\partial z\,\partial y\,\partial x}u(x,y,z), \frac{\partial^4}{\partial z^3\,\partial x}u(x,y,z)\right]$$

```
> SolSeries(J,5);
Error, (in SolSeries) invalid point; solution of, y = 0
```

Wir können in der Tat nicht mehr um (0,0,0) herum entwickeln. Es gibt aber die Möglichkeit einen anderes Zentrum für die Entwicklung anzugeben:

```
> SolSeries(J,5,[0,1,0]);
```

$$\Big[u(x,y,z) = C1_{0,0,0} + C1_{1,0,0}\,x + C1_{0,1,0}\,(y-1) + C1_{0,0,1}\,z - \frac{1}{2}C1_{0,0,2}\,x^2 + C1_{1,1,0}\,x\,(y-1) + C1_{1,0,1}\,x\,z + C1_{0,1,1}\,(y-1)\,z + \frac{1}{2}C1_{0,0,2}\,z^2 - \frac{1}{6}C1_{1,0,2}\,x^3 - \frac{1}{2}C1_{0,0,3}\,x^2\,z + C1_{1,1,1}\,x\,(y-1)\,z + \frac{1}{2}C1_{1,0,2}\,x\,z^2 + \frac{1}{2}C1_{0,0,2}\,(y-1)\,z^2 + \frac{1}{6}C1_{0,0,3}\,z^3 - \frac{1}{6}C1_{1,0,3}\,x^3\,z + \frac{1}{2}C1_{1,0,2}\,x\,(y-1)\,z^2 + \frac{1}{6}C1_{1,0,3}\,x\,z^3 + \frac{1}{6}C1_{0,0,3}\,(y-1)\,z^3 + \frac{1}{6}C1_{1,0,3}\,x\,(y-1)\,z^3\Big]$$

Vielleicht haben wir Glück, daß wir viele Polynomlösungen haben, zumal keine Terme der Ordnun bei den Anfangstermen der Lösungen auftauchten:

```
> PolySol(J,5,[0,1,0],‘b‘);
```

$$\Big[u(x,y,z) = (C1_{1,0,1} - C1_{1,1,1})\,x\,z + \frac{1}{6}C1_{1,0,3}\,x\,z^3\,y + (C1_{0,0,1} - C1_{0,1,1})\,z - \frac{1}{6}C1_{1,0,2}\,x^3 - \frac{1}{2}C1_{0,0,2}\,x^2 - \frac{1}{2}C1_{0,0,3}\,x^2\,z + \frac{1}{2}C1_{0,0,2}\,z^2\,y + C1_{1,1,1}\,x\,z\,y - \frac{1}{6}C1_{1,0,3}\,x^3\,z + C1_{1,1,0}\,x\,y + \frac{1}{2}C1_{1,0,2}\,x\,z^2\,y + \frac{1}{6}C1_{0,0,3}\,z^3\,y + C1_{0,1,1}\,z\,y - C1_{0,1,0} + C1_{0,0,0} + (-C1_{1,1,0} + C1_{1,0,0})\,x + C1_{0,1,0}\,y\Big]$$

```
> b;
```

$$\Big[[u(x,y,z)=1], [u(x,y,z)=x], [u(x,y,z)=y-1], [u(x,y,z)=z], \left[u(x,y,z) = -\frac{1}{2}x^2 + \frac{1}{2}y\,z^2\right], [u(x,y,z)=x\,y-x], [u(x,y,z)=x\,z], [u(x,y,z)=-z+y\,z], \left[u(x,y,z) = -\frac{1}{6}x^3 + \frac{1}{2}x\,y\,z^2\right], \left[u(x,y,z) = -\frac{1}{2}x^2\,z + \frac{1}{6}y\,z^3\right], [u(x,y,z)=-x\,z+x\,y\,z], \left[u(x,y,z) = \frac{1}{6}x\,y\,z^3 - \frac{1}{6}x^3\,z\right]\Big]$$

```
> nops(b);
```

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Vielleicht brauchen wir bei einer anderen Ordnung nicht durch y zu teilen:

```
> Jn:=JanetBasis(L,ivar,dvar,[z,y,x]);
```

$$Jn := \left[\left[\frac{\partial^2}{\partial y^2}u(x,y,z), \left(\frac{\partial^2}{\partial z^2}u(x,y,z)\right) + y\left(\frac{\partial^2}{\partial x^2}u(x,y,z)\right), -\left(\frac{\partial^3}{\partial y\,\partial x^2}u(x,y,z)\right), \frac{\partial^3}{\partial z\,\partial y^2}u(x,y,z),\right.\right.$$

$$\frac{\partial^4}{\partial x^4}\mathrm{u}(x,y,z), -\left(\frac{\partial^4}{\partial z\,\partial y\,\partial x^2}\mathrm{u}(x,y,z)\right), \frac{\partial^5}{\partial z\,\partial x^4}\mathrm{u}(x,y,z)\Bigg], [x,y,z], [u], [z,y,x]\Bigg]$$

```
> ZeroSets();
```

$$[\,]$$

```
> HilbertSeries(t);
```

$$1+3\,t+4\,t^2+3\,t^3+t^4$$

Also hatten wir in der Tat alle Lösungen.

```
>
```