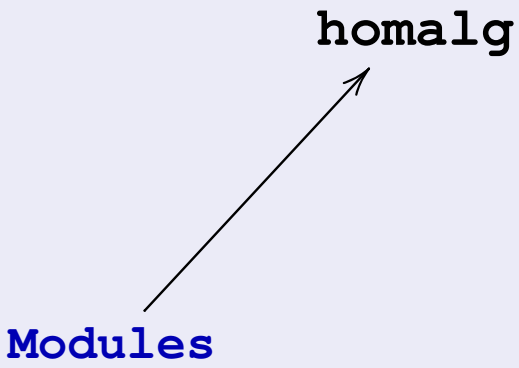
The homalg project and its related packages

The homalg project authors

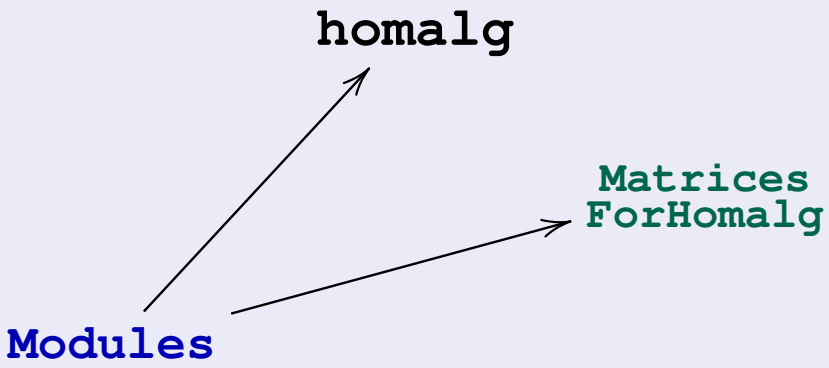
2007-2012

homalg

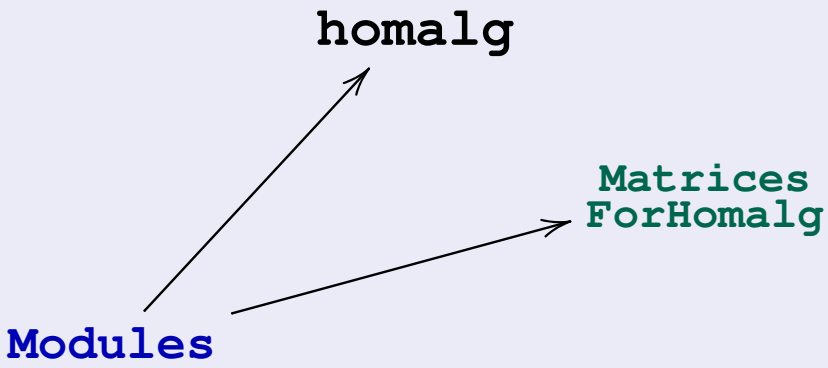
The category of finitely presented modules as the basic example of a computable ABELian category



Matrices provide the needed data structure for finitely presented modules and their morphisms

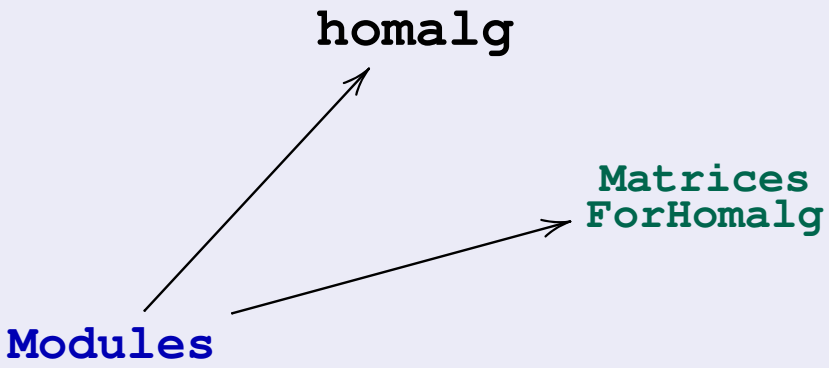


Candidates: There are several systems that could host homalg



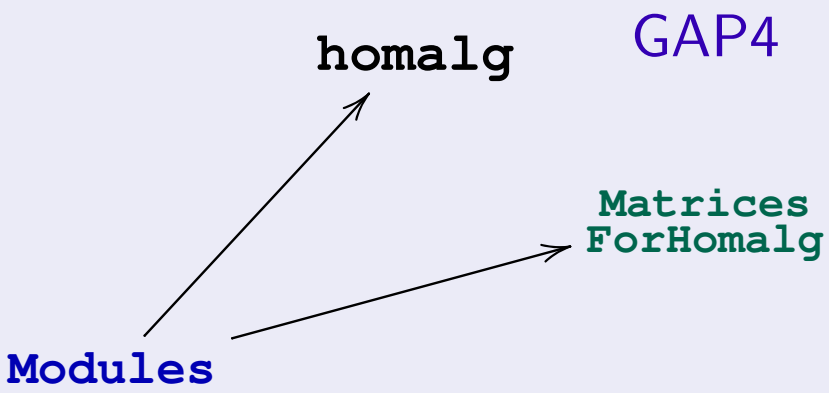
Maple MAGMA Macaulay2 Sage GAP SINGULAR

Candidates: There are several systems that could host `homalg`, each supporting certain kinds of rings



Maple	MAGMA	Macaulay2	Sage	GAP	SINGULAR
$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$\mathbb{Z}[x,\partial]$, $\ldots$	$\mathbb{Z}[x]$, $\mathbb{F}[x]$	$\mathbb{F}[x]$, $\mathbb{F}[x,\partial]$, $\ldots$	$\mathbb{Z}$, $\ldots$	$\mathbb{Z}$	$\mathbb{F}[x]$, $\mathbb{F}[x,\partial]$, $\ldots$

GAP4: The best suited *language* for abstract mathematical programming



Maple
$\Downarrow$
$\mathbb{Z}[x, \partial]$, ...

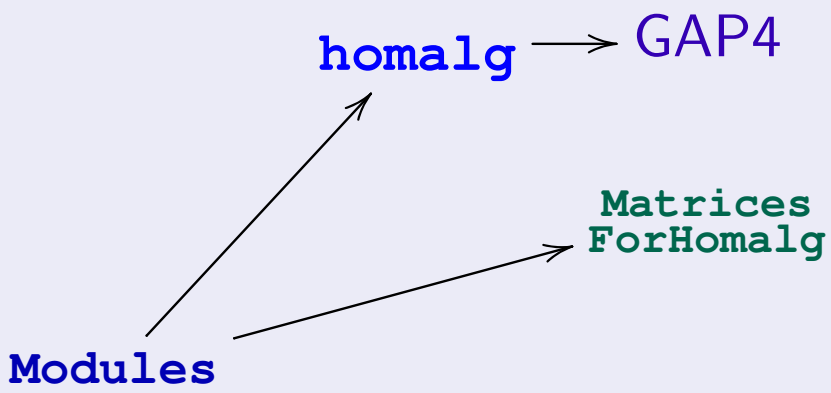
MAGMA
$\Downarrow$
$\mathbb{Z}[x]$, $\mathbb{F}[x]$

Macaulay2
$\Downarrow$
$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...

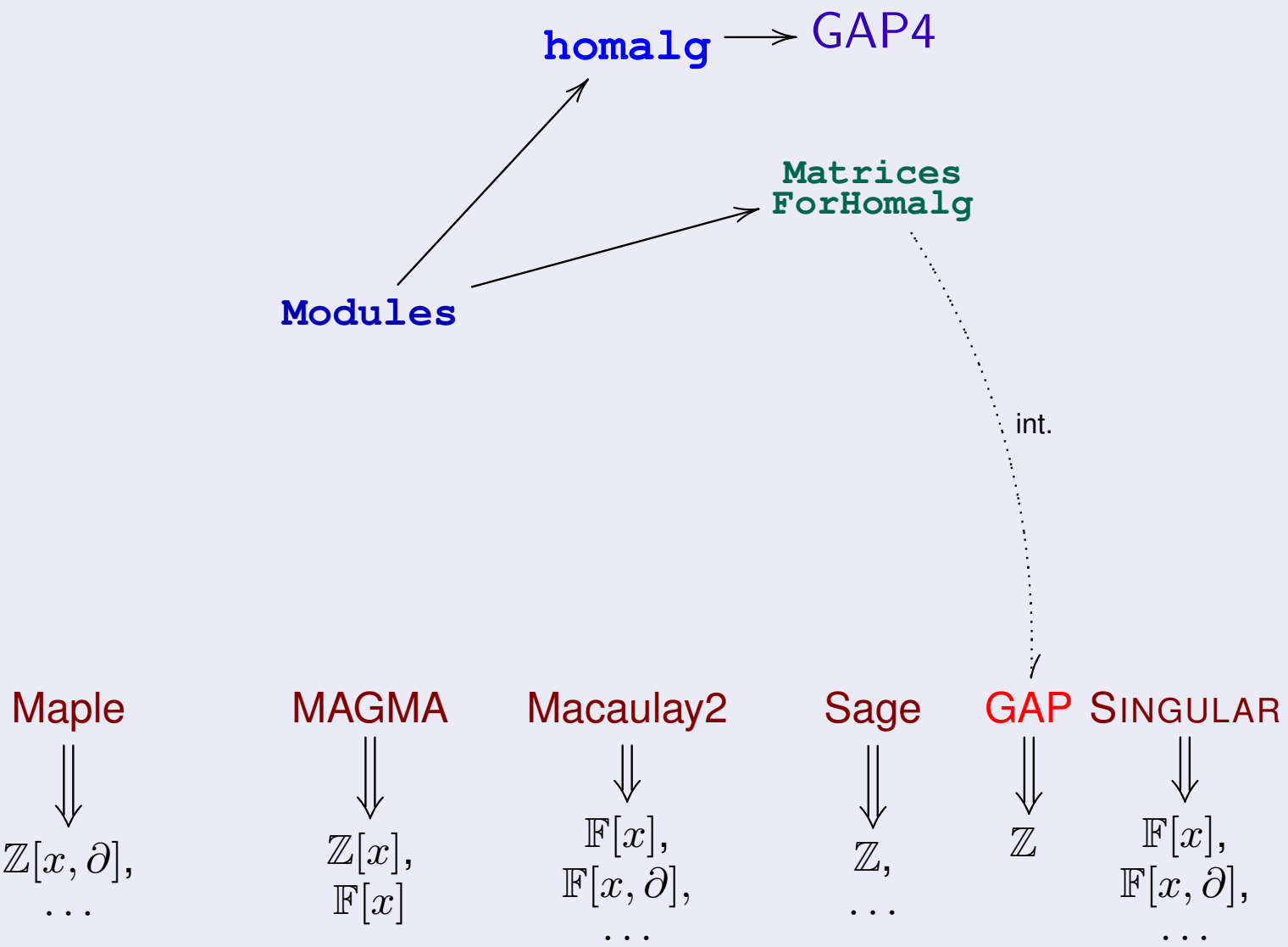
Sage
$\Downarrow$
$\mathbb{Z}$, ...

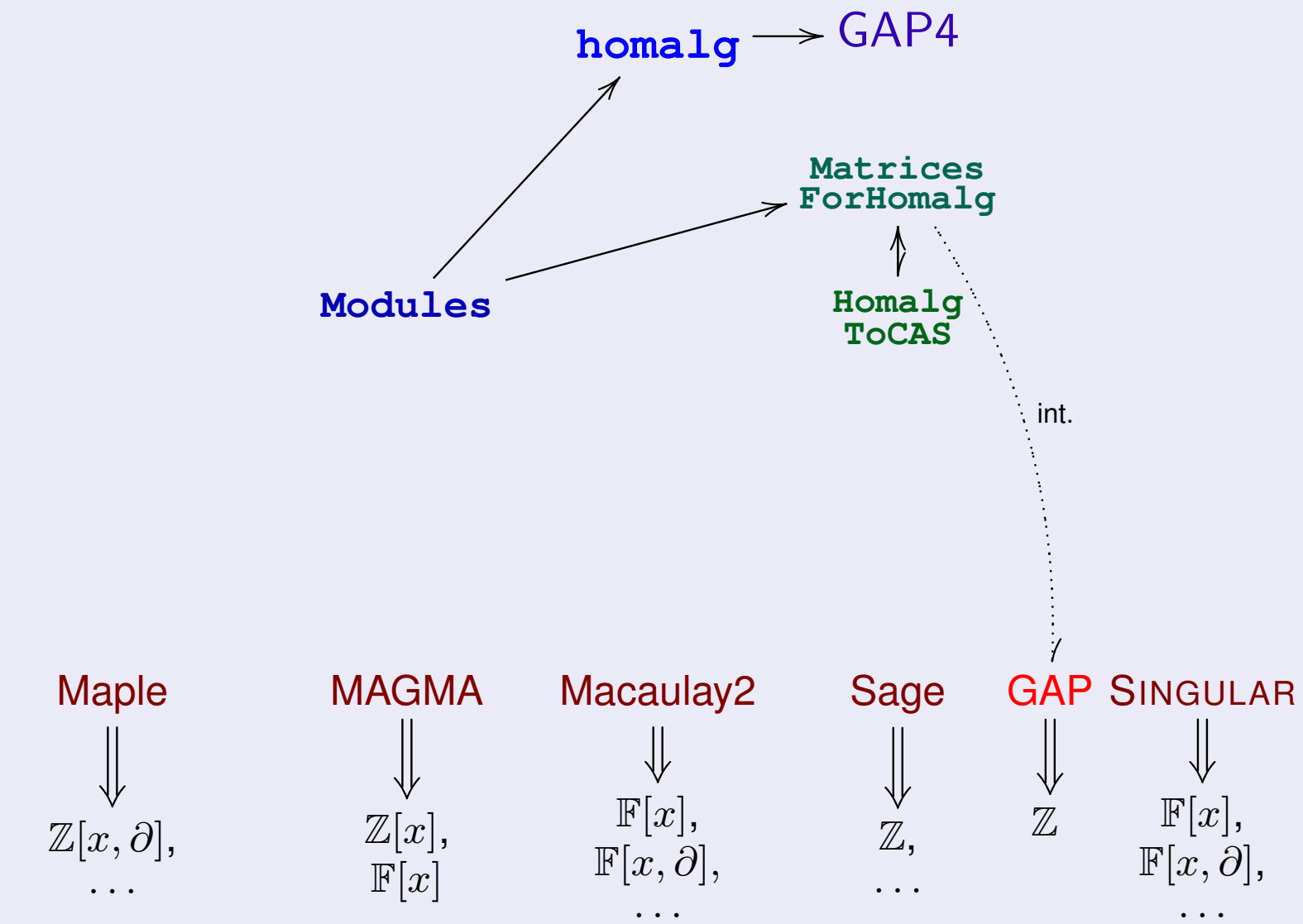
GAP
$\Downarrow$
$\mathbb{Z}$

SINGULAR
$\Downarrow$
$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...



Maple	MAGMA	Macaulay2	Sage	GAP	SINGULAR
$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$\mathbb{Z}[x,\partial]$, $\ldots$	$\mathbb{Z}[x]$, $\mathbb{F}[x]$	$\mathbb{F}[x]$, $\mathbb{F}[x,\partial]$, $\ldots$	$\mathbb{Z}$, $\ldots$	$\mathbb{Z}$	$\mathbb{F}[x]$, $\mathbb{F}[x,\partial]$, $\ldots$


homalg
GAP4
Matrices
ForHomalg
Modules
int.
Maple
MAGMA
Macaulay2
Sage
GAP
SINGULAR
$\mathbb{Z}[x, \partial]$, ...
$\mathbb{Z}[x]$, $\mathbb{F}[x]$
$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...
$\mathbb{Z}$, ...
$\mathbb{Z}$
$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...


homalg
GAP4
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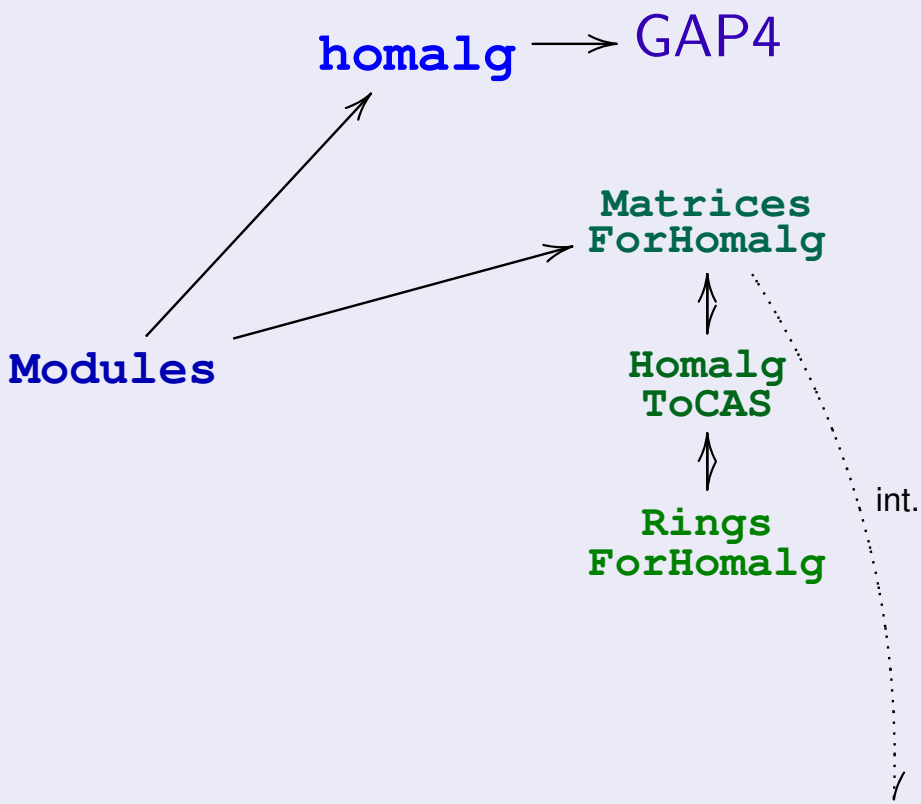
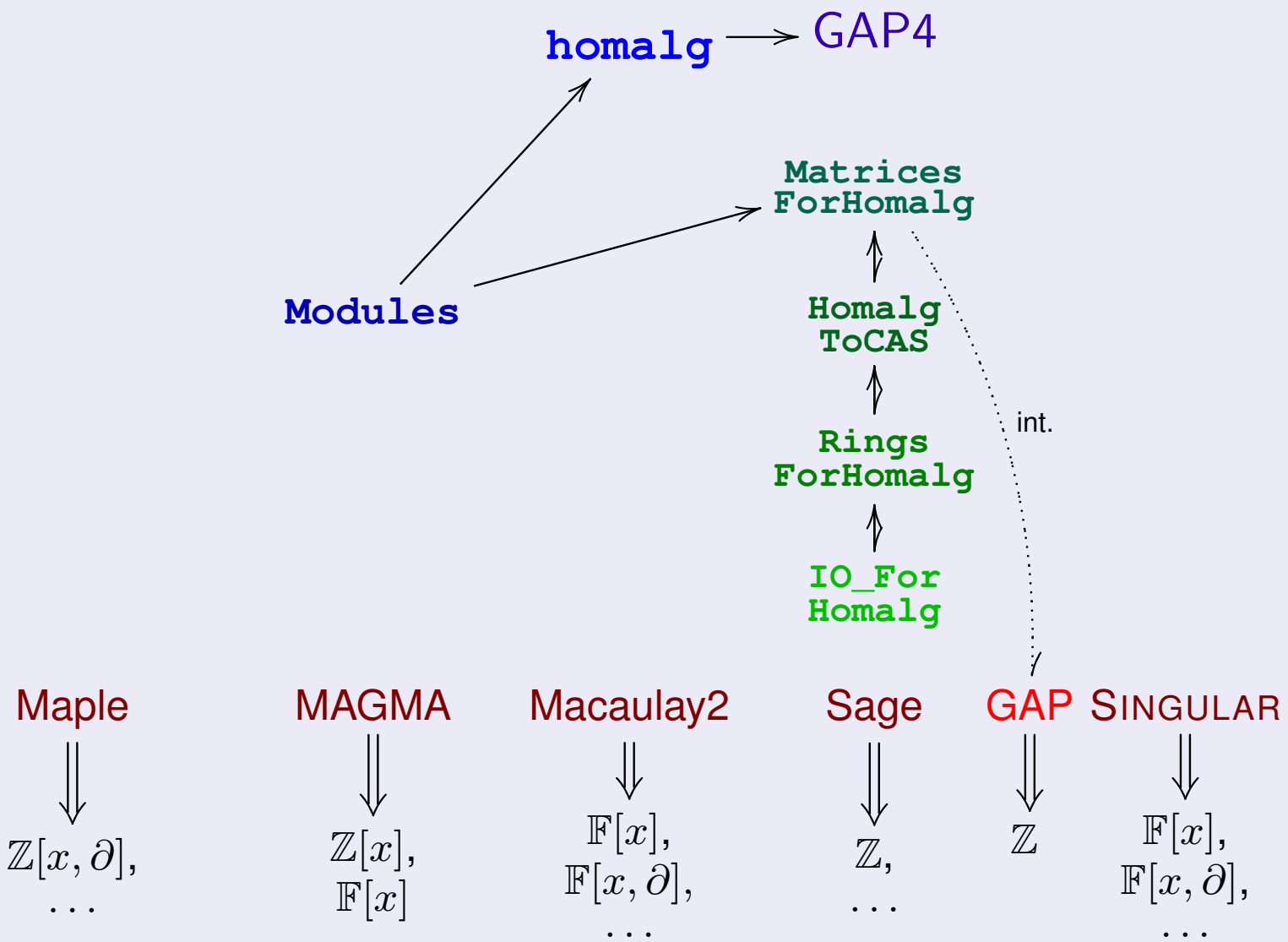
$\mathbb{Z}$, ...
$\mathbb{Z}$
$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...

RingsForHomalg: The dictionaries used by MatricesForHomalg



Maple	MAGMA	Macaulay2	Sage	GAP	SINGULAR
$\mathbb{Z}[x, \partial]$, ...	$\mathbb{Z}[x]$, $\mathbb{F}[x]$	$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...	$\mathbb{Z}$, ...	$\mathbb{Z}$	$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...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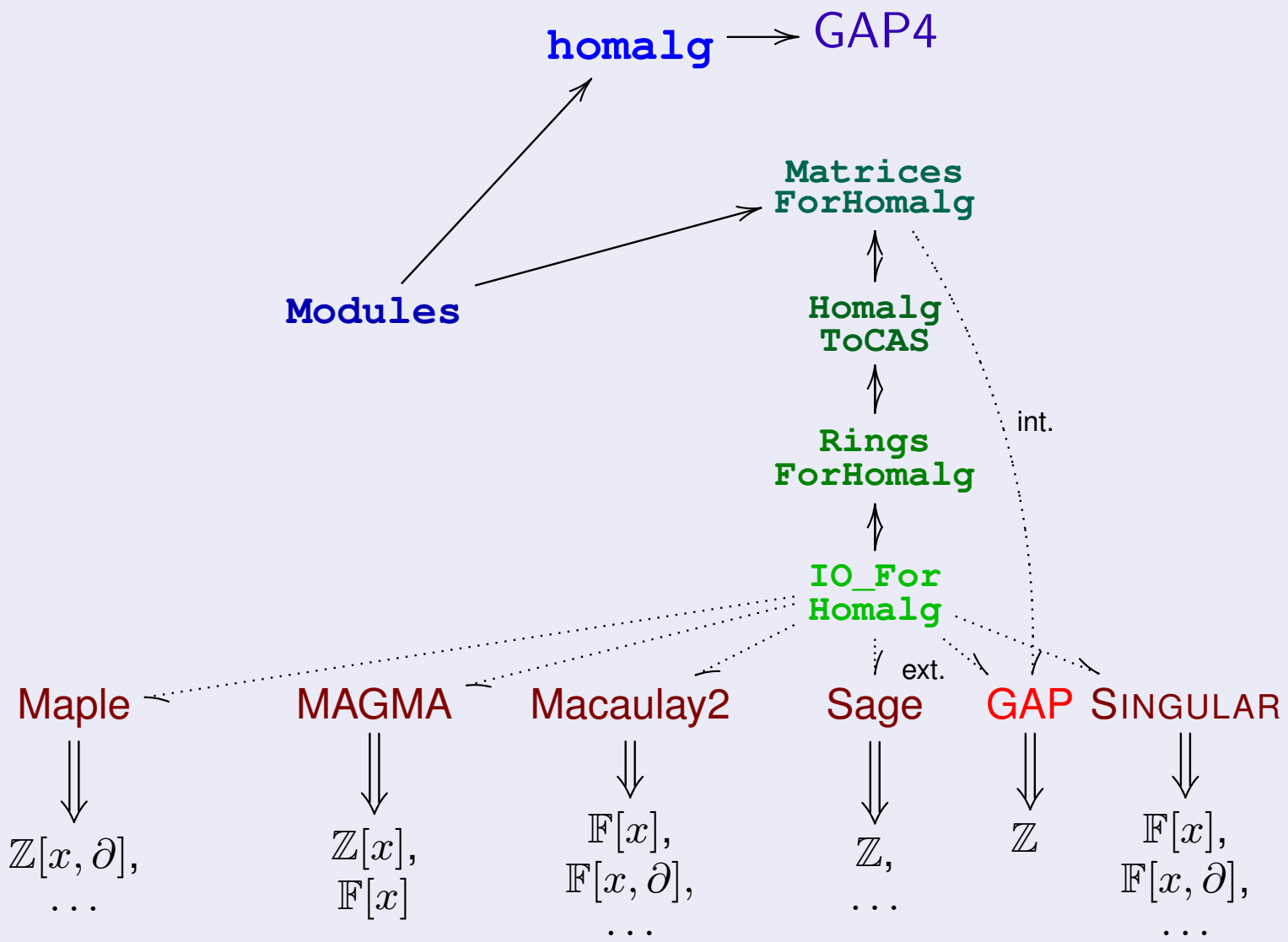
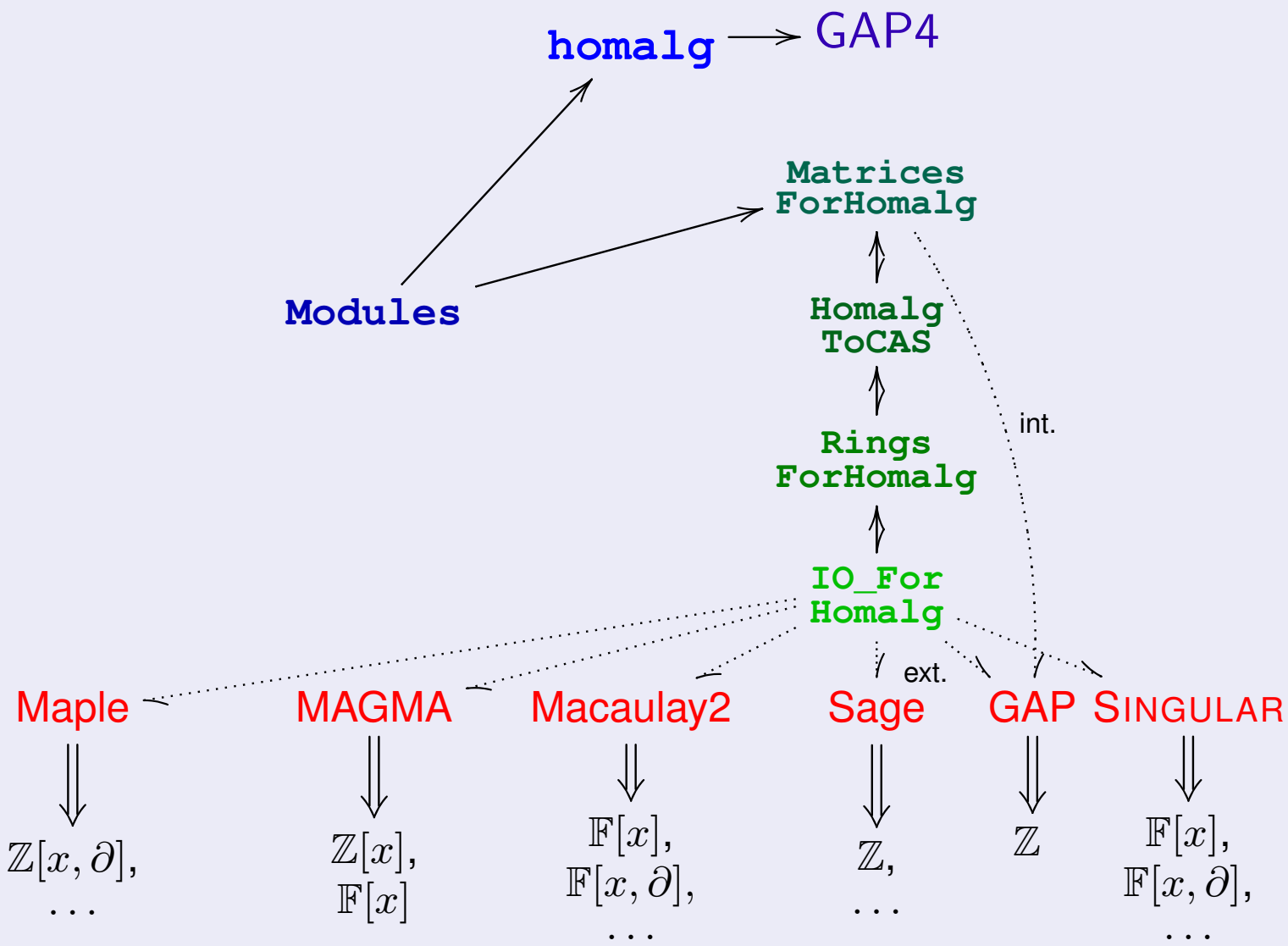
$\mathbb{Z}$, ...
$\mathbb{Z}$
$\mathbb{F}[x]$, $\mathbb{F}[x, \partial]$, ...

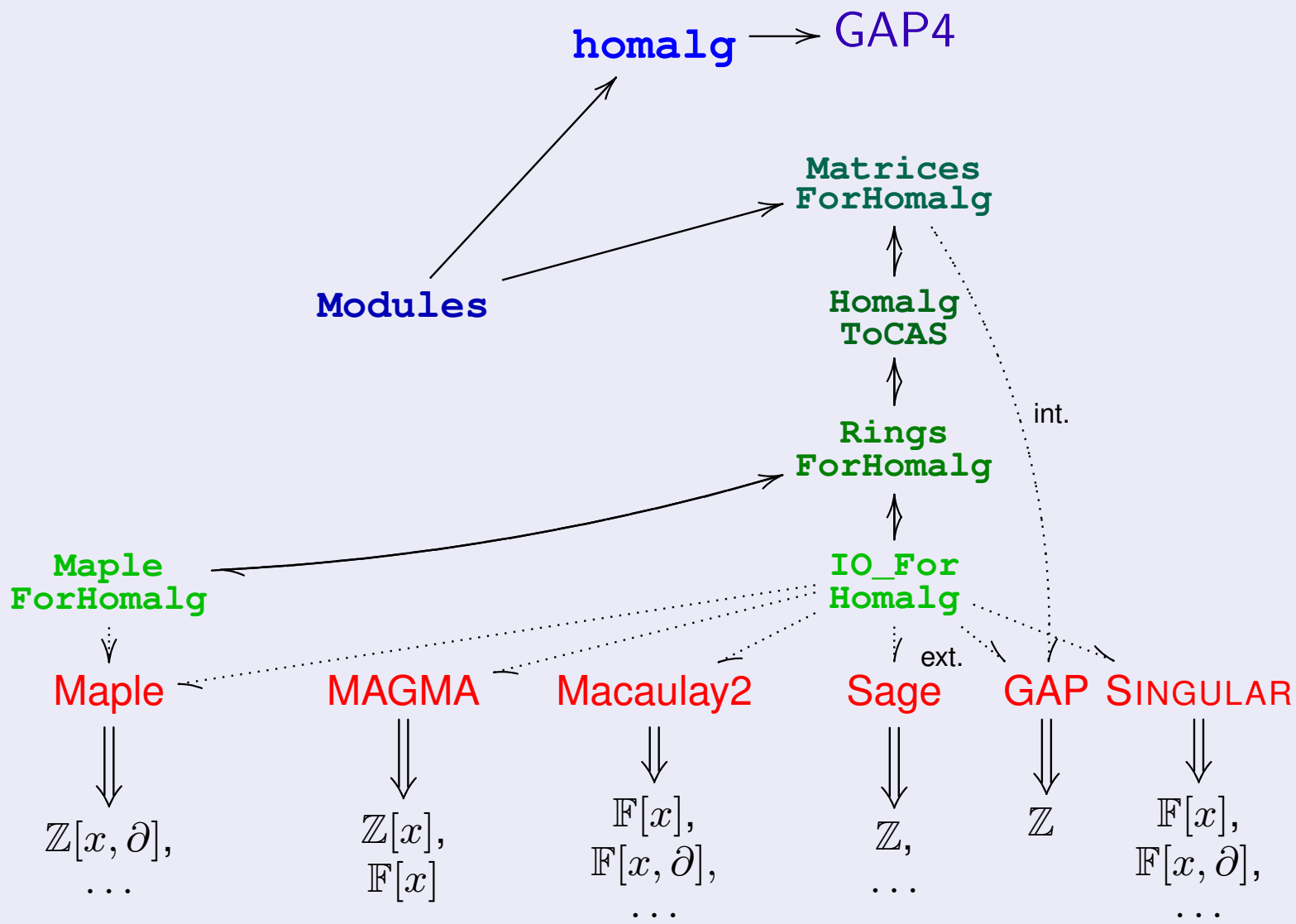
IO_ForHomalg: Communicate via streams with various CA systems through their command line interface



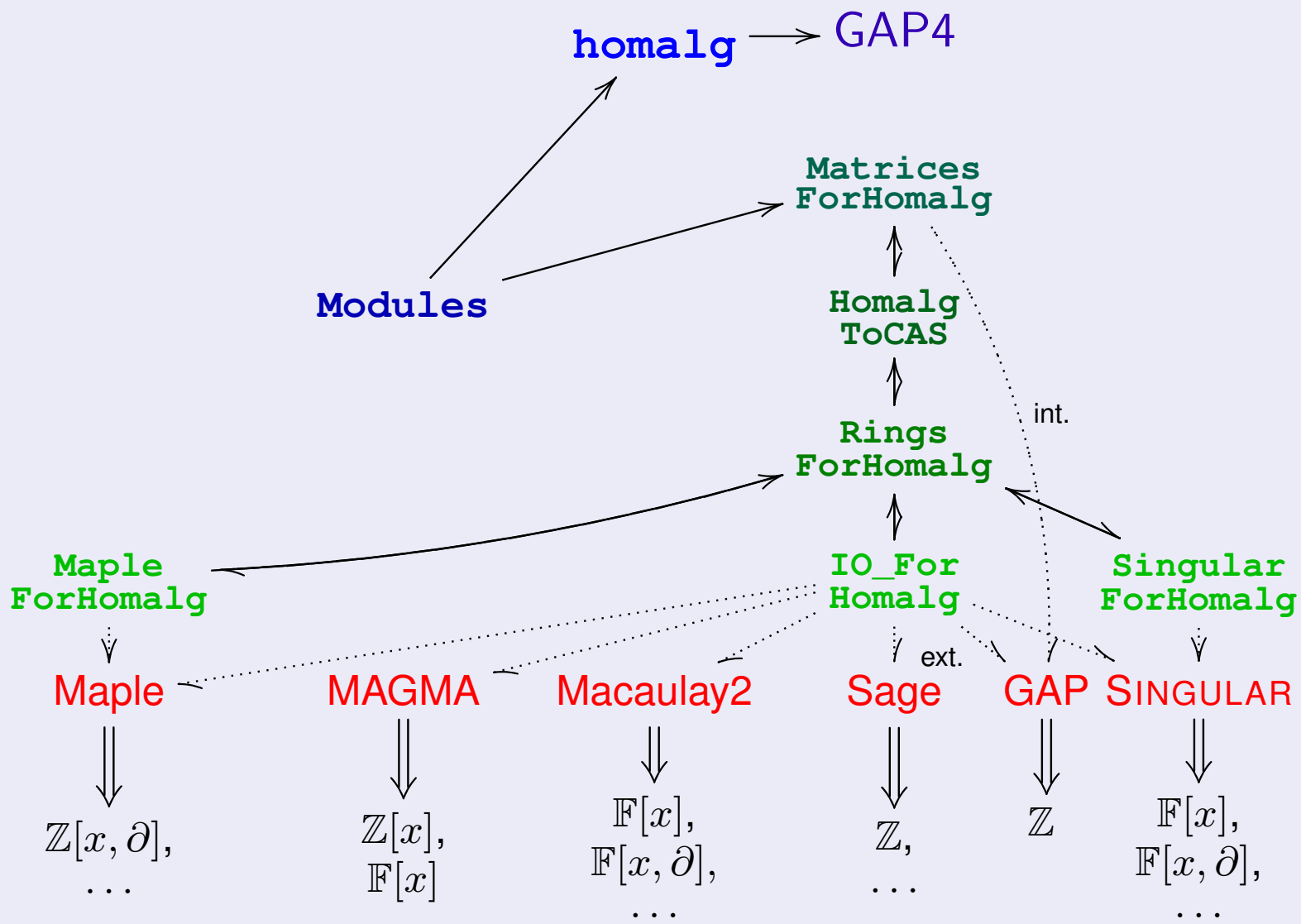
External CASs host the matrices and GAP4 contains the higher logic → *Principle of least communication*



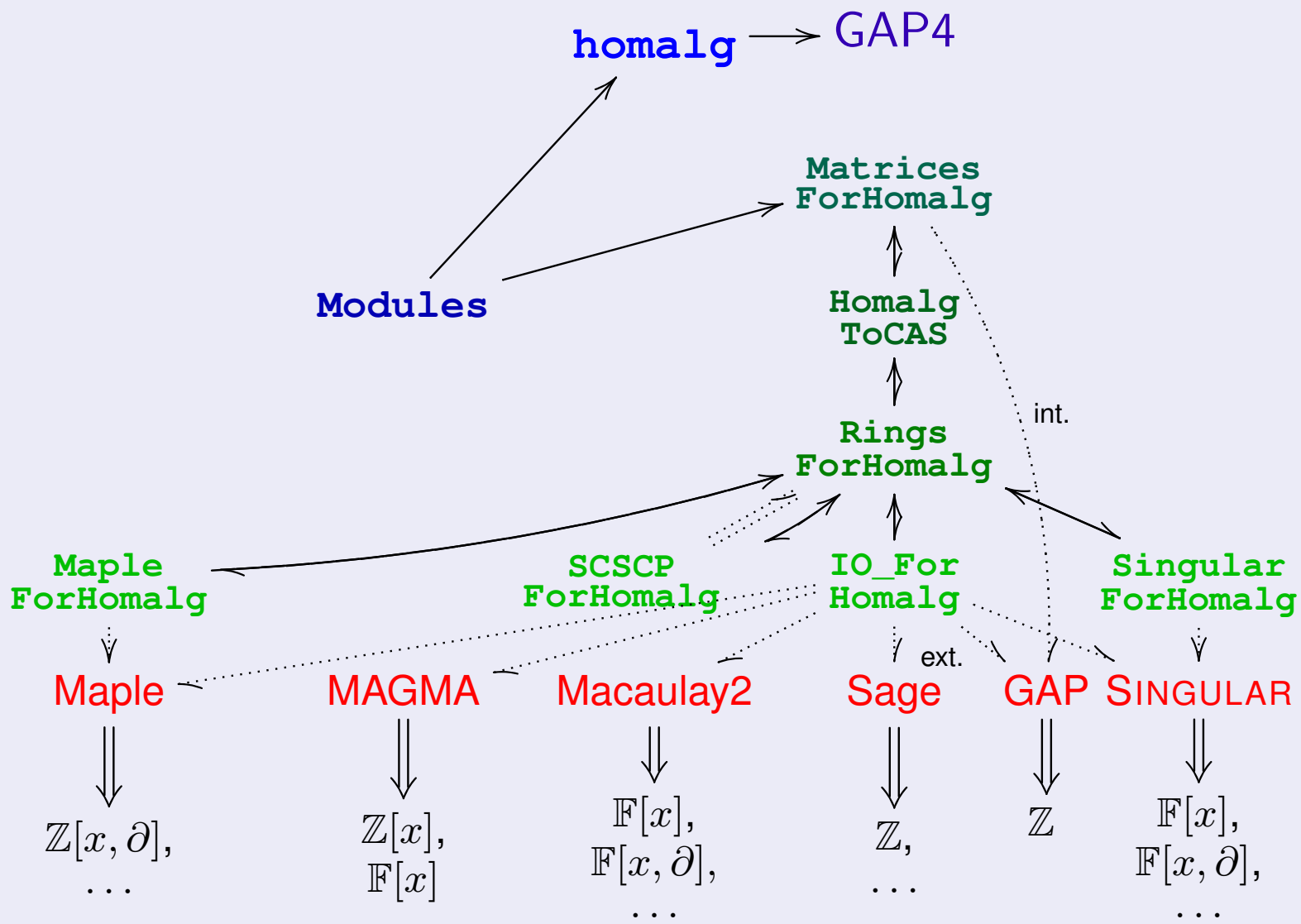
MapleForHomalg: Communicate with Maple's interpreter, shortcutting its command line interface



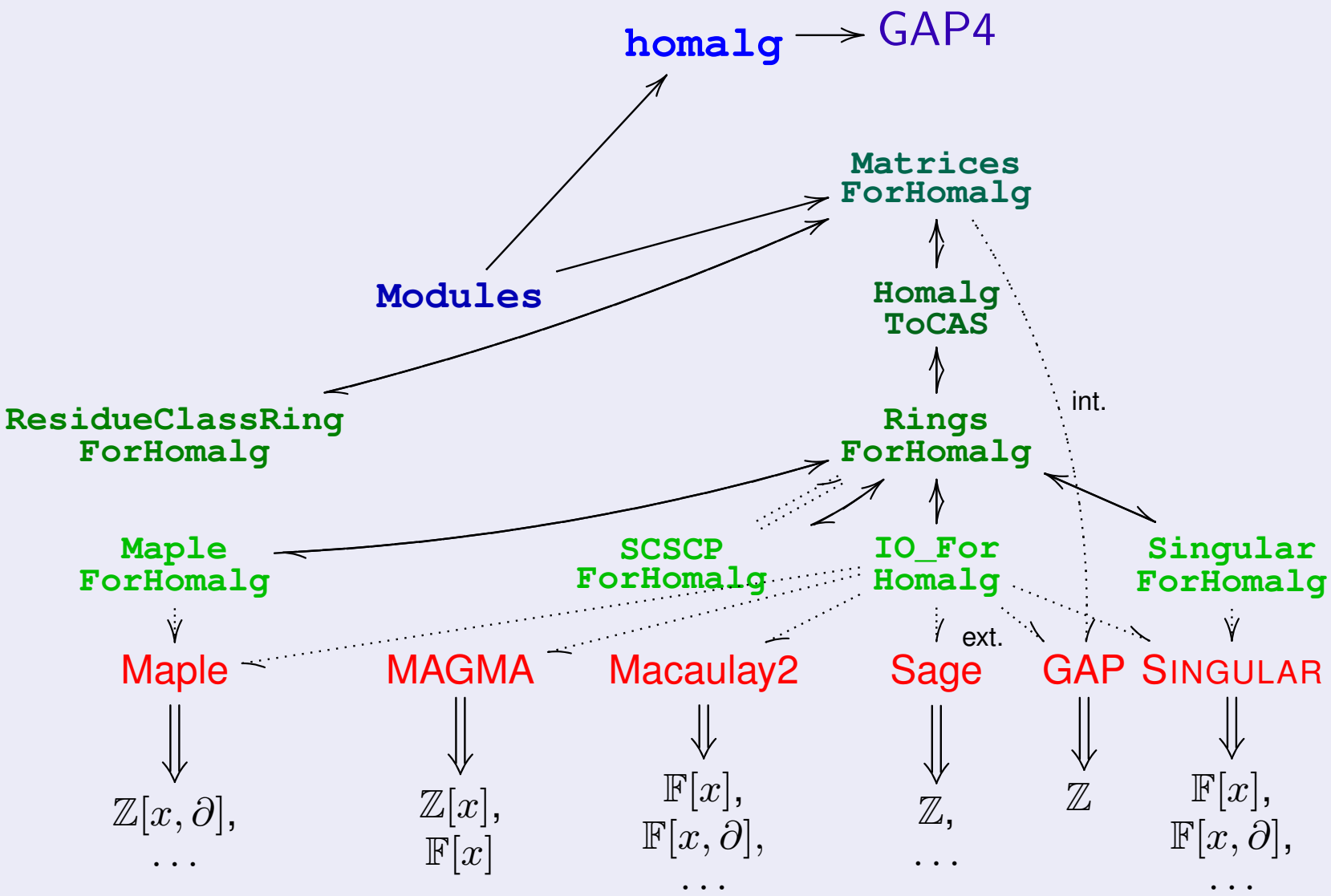
Future: Communicate with interpreters of various CASs shortcutting their command line interface.



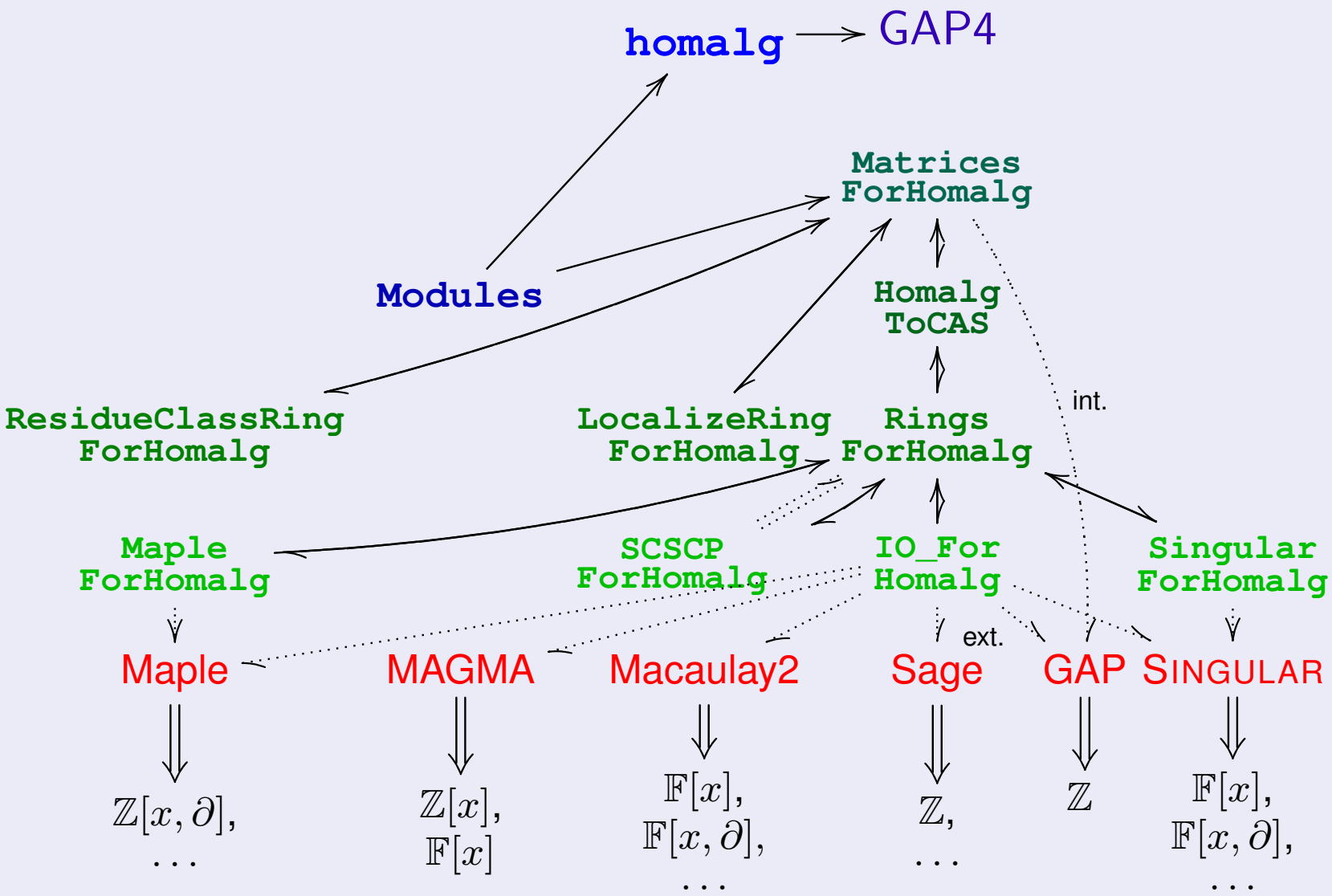
SCSCP_ForHomalg: SCSCP can be used to reach computing engines running on local or remote machines



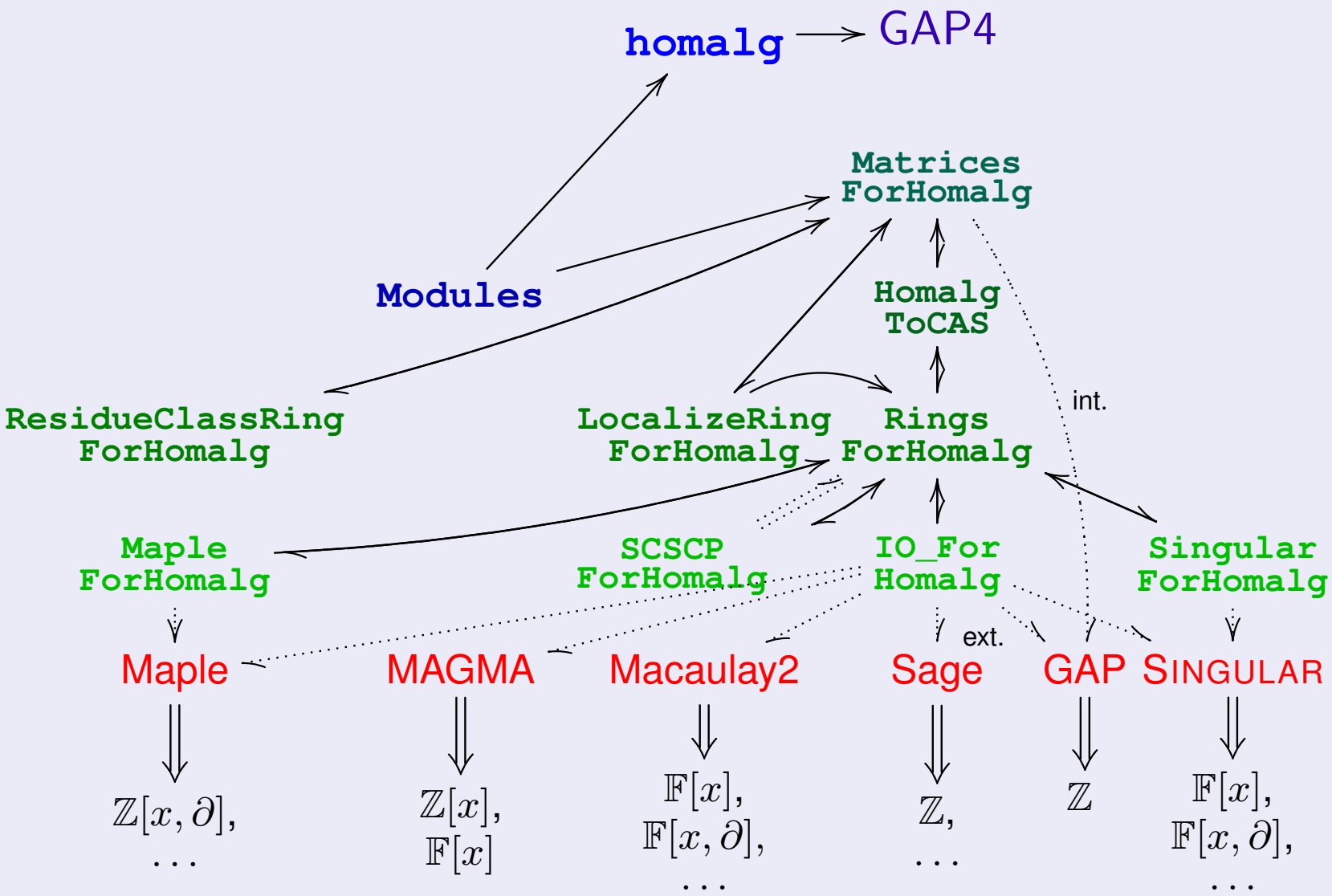
ResidueClassRingForHomalg: Support for residue class rings



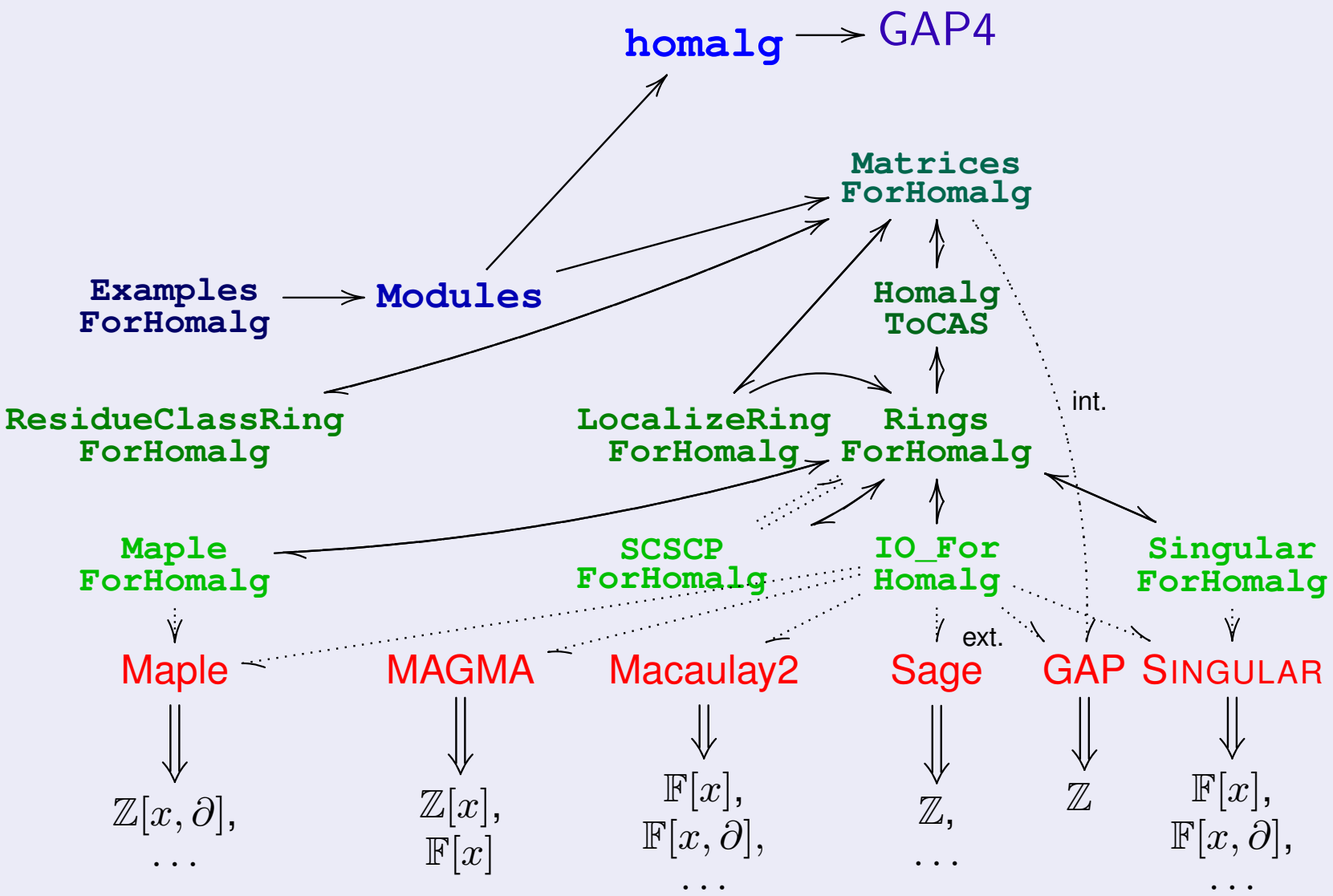
LocalizeRingForHomalg: Localizations of commutative rings in homalg at maximal ideals.

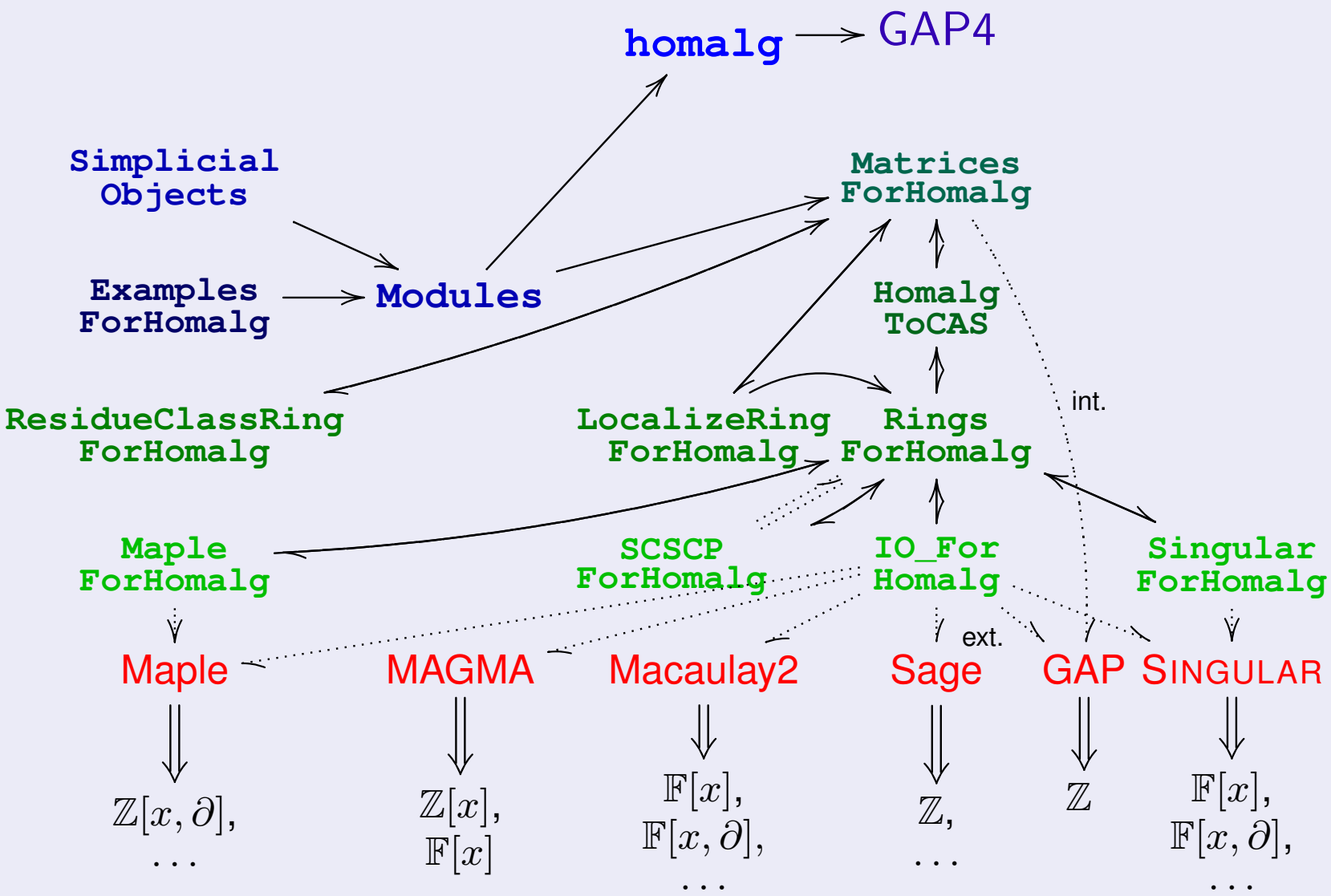


LocalizeRingForHomalg: Use MORA's algorithm in SINGULAR to localize polynomial rings at maximal ideals.



ExamplesForHomalg: Computing-engine-independent example using homalg.




homalg
GAP4
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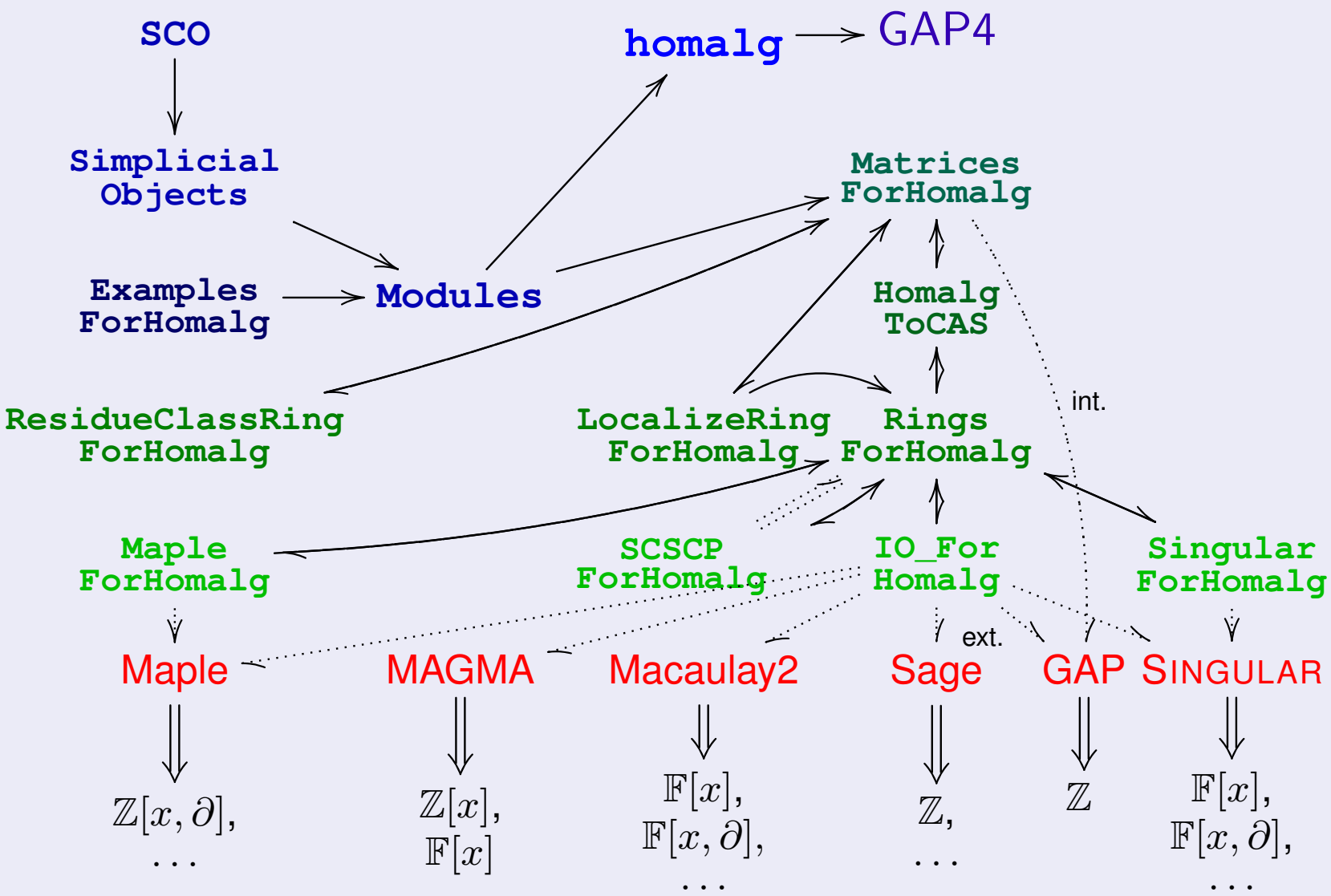
SCO
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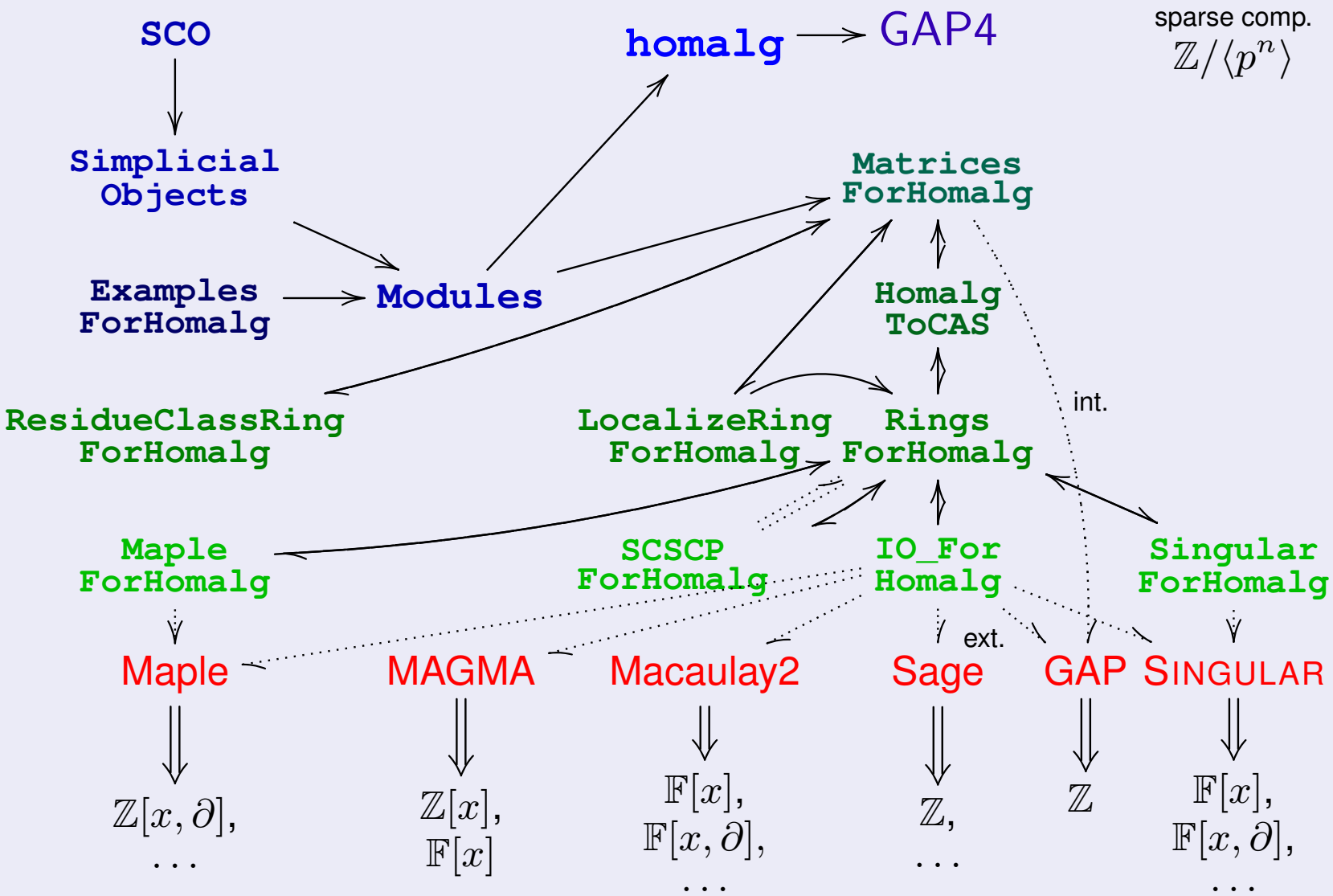
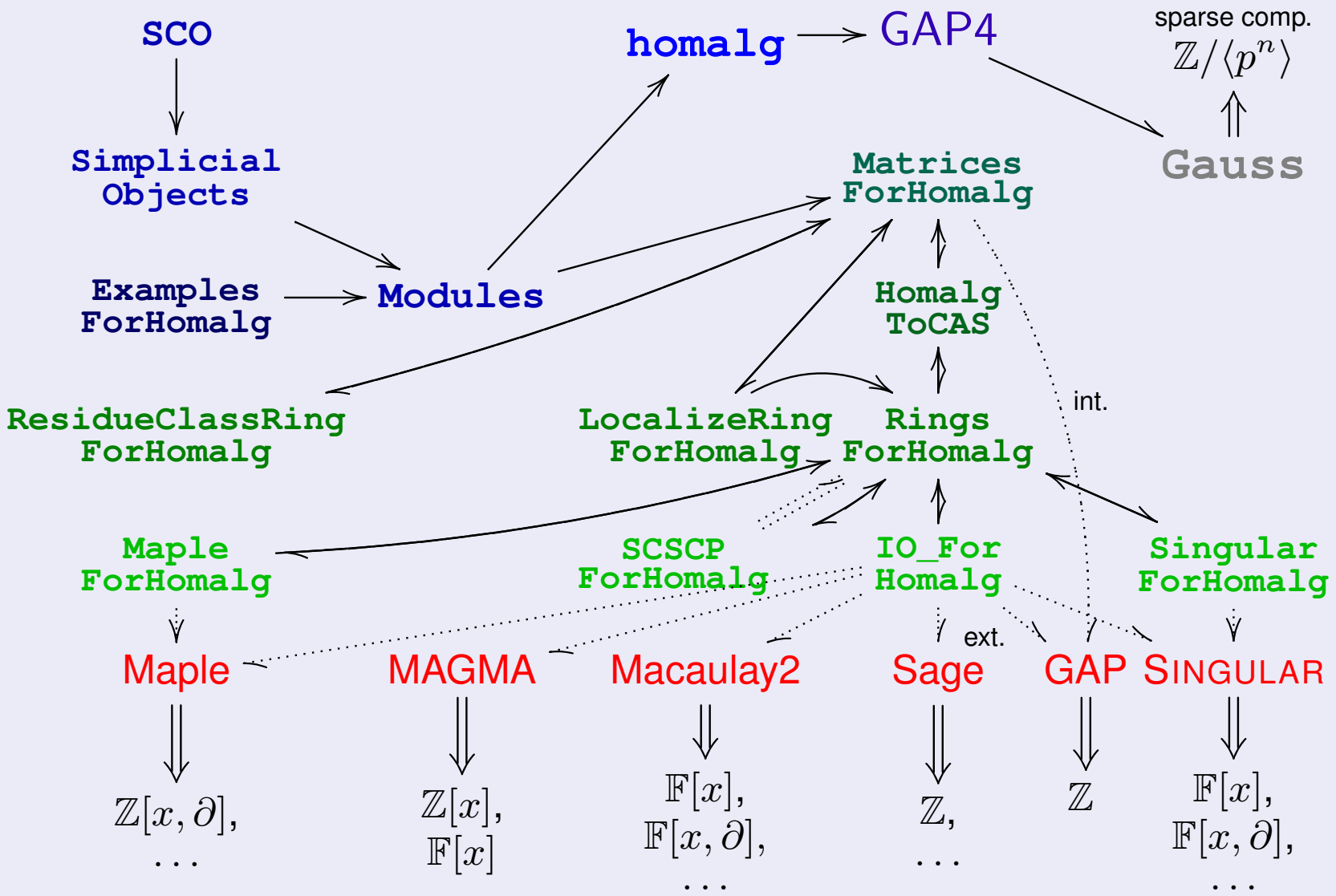
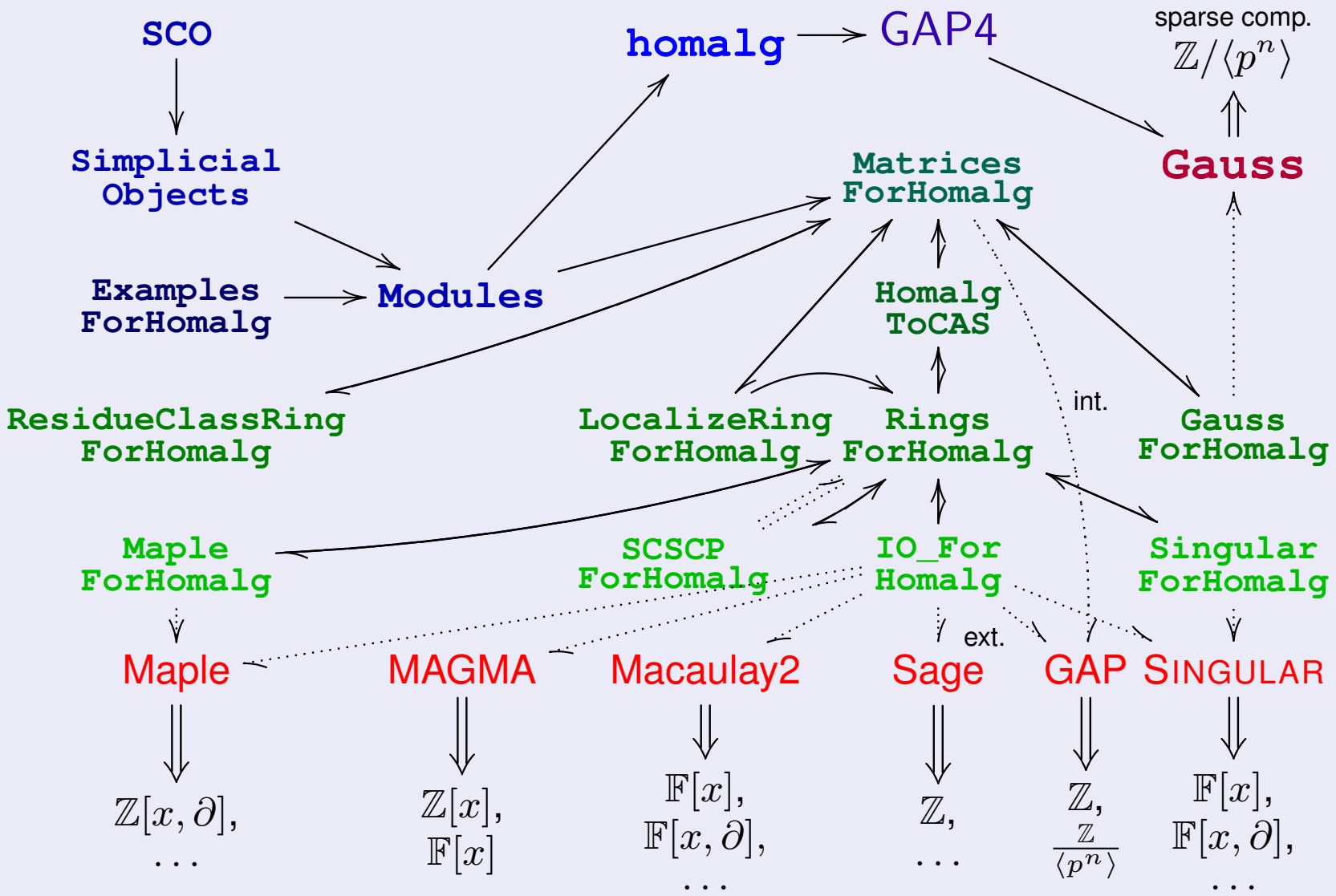
Sparse computations over p-adic numbers (a necessity not only for SCO)



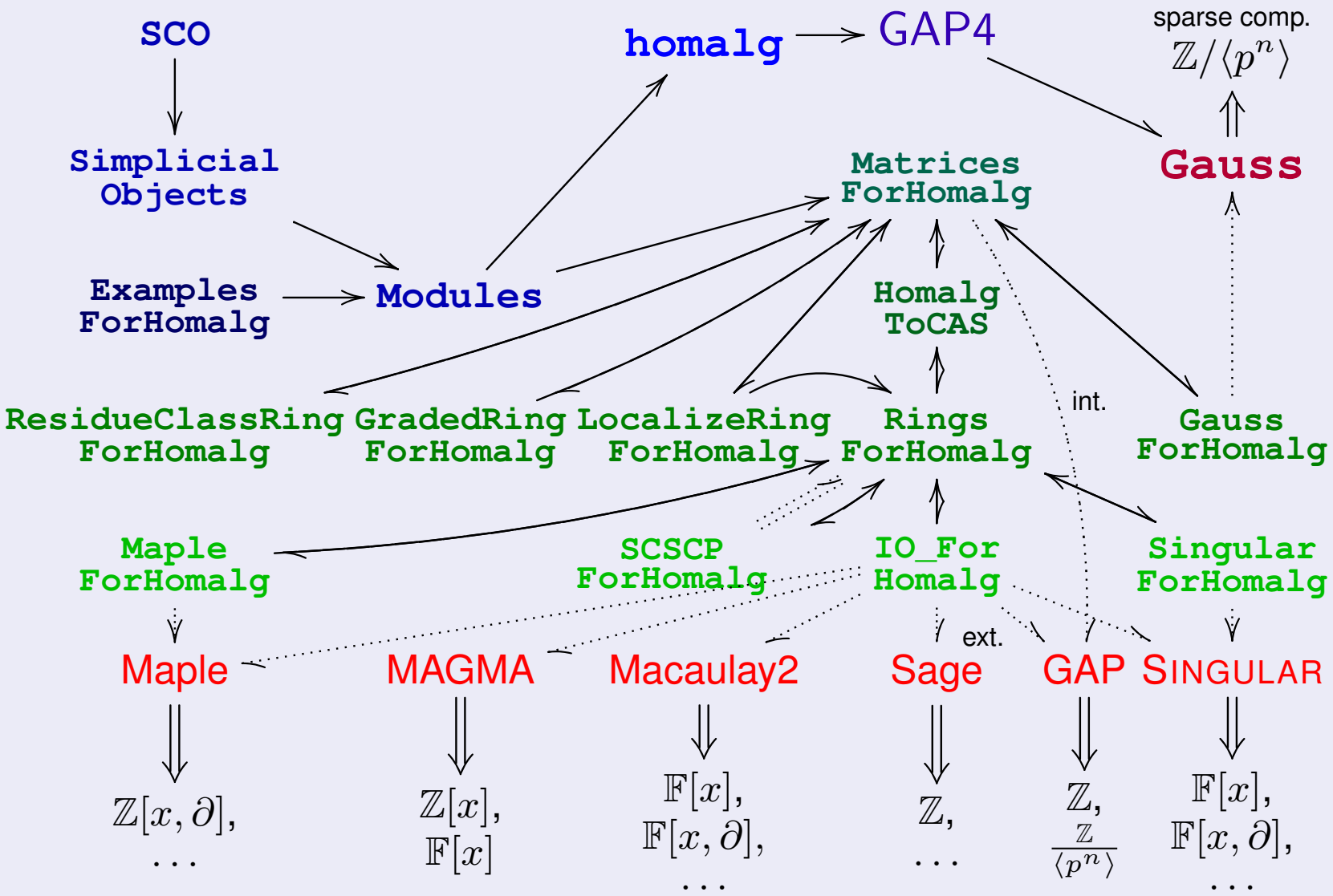
Gauss: Added missing RREF to GAP4 for **sparse** matrices over $\mathbb{Z}/p^n\mathbb{Z}$ and $\mathbb{Q}$



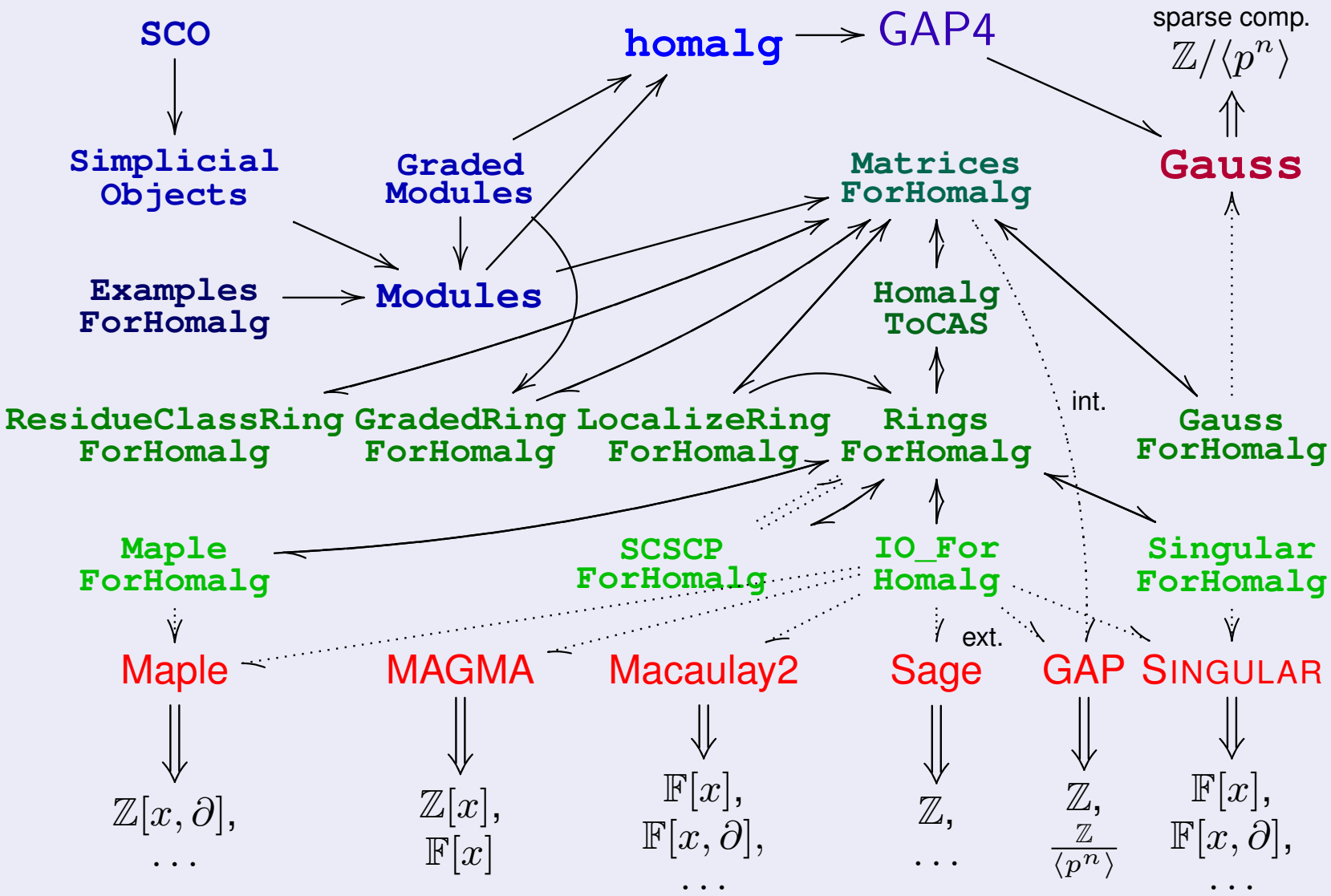
GaussForHomalg: Linking Gauss and homalg



GradedRingForHomalg: Multi-graded rings serve as the data structure underlying many geometric constructions



GradedModules: Graded modules over multi-graded rings



Sheaves: Coherent sheaves of modules (& future projects: Advanced applications building upon homalg)